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## PICATINNY ARSENAL

TECHNICAL REPORT 3059

# A UNIQUE UNIVERSAL TYPE INSTRUMENT

## TO

## LOCATE CENTER OF GRAVITY

## OF

## VARIOUS WARHEADS

BY

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PICATINNY ARSENAL

DOVER, NEW JERSEY

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# ABSTRACT

A unique, universal type device to locate the center of gravity of various warheads is described. Simple basic principles are utilized in a new configuration to produce a highly precise instrument which may be adapted to locate center of gravity of warheads with a constant accuracy of about 0.002 inch. Both longitudinal distance from warhead reference surface and radial displacement from warhead axis are determined in a single handling of the warhead, thereby eliminating the need for two separate devices.

The instrument was designed specifically for warheads in the Little-john and Honest John Systems (weight range up to 1700 pounds; centroid up to 37 inches from rear mounting surface). However, its total range of capability extends from essentially zero to a warhead which exerts 80,000 inch-pounds of moment (warhead weight times centroidal distance from rear reference plane).

Mounting provisions are made for the following items of current production: Littlejohn Warheads M146 and M8, and Honest John Warheads M38, M132, and M144.

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## CONCLUSIONS AND RECOMMENDATIONS

Evaluation of this instrument through performance of a series of calibrating and use tests have proved that its capabilities exceed initial design objectives by a factor of five in accuracy. All values required to be determined are readily measurable through use of the instrument.

The basic principle of the design, suspension of the total mass from a single pivot point, is shown to be a highly effective method of attaining the desired results. It is felt that the overall design, with its great flexibility, utility and accuracy, represents a significant advance to the state of the art. Its representation in the presently existing prototype is a highly useful device.

The authors recommend consideration of extension of the experience gained here to design of more sophisticated instruments. For example, one could visualize an instrument which would have balancing weights whose positions are controlled by a servo system, with automatic readout of net moment required to reach balance. An instrument could be readily designed to be self-balancing, with the pivot being adjustable to the extent that the completely unloaded instrument could be leveled, thus eliminating the need for any standard such as is required for the existing prototype instrument. A further possibility would eliminate any balancing weights, accomplishing balance by means of a movable pivot whose position longitudinally and radially would establish the warhead center of gravity position.

Determination of the weight of the warhead is beyond the function of the present instrument, and thus subject to unknown variations. Incorporation of a load cell into the fulcrum support would make the instrument fully self-contained and independent of external weighing means, and would assure weight (and thus center of gravity location) determination within known uncertainty limits. Then the operating instructions and acceptance limits can take this known uncertainty into account to assure acceptance of only good units.

Based on the experience gained from construction and testing of the prototype instrument, a redesign effort is currently in process. This will provide:

2.

1. Simplified construction details in the areas of vernier adjustments and side-sway control.

2. Reduced weight and elimination of all end-play in the bearing assembly.

3. Built-in facilities for checking and setting true vertical alignment, and

4. The integral weight measuring load cell noted above.

## SYMBOLS, DEFINITIONS AND CONVENTIONS

1. **Symbols**

   $k$ - Distance from pivot axis to face plate, measured along x-axis

   $M$ - Total longitudinal restoring moment

   $M_1$ - Front balancing moment = $W_1 s_1$

   $M_2$ - Rear balancing moment = $W_2 s_2$

   $M_r$ - Radial restoring moment

   $M_v$ - Moment exerted by 1 pound vernier weight at rear of instrument, equal in inch-pounds to pointer reading on scale.

   $\bar{R}$ - Radial displacement of warhead CG from warhead axis

   $s_1$ - Distance from pivot to front balancing weight CG

   $s_2$ - Distance from pivot to rear balancing weight CG

   $W$ - Weight of warhead

   $W_1$ - Balancing weight added to front

   $W_2$ - Balancing weight added to rear

   $W_A$ - Weight of special adaptor to mount particular warhead

   $W_F$ - Net fixture weight (does not include weight of pivot cage)

   $\bar{X}$ - Longitudinal location of warhead center of gravity, measured from design reference point on warhead

   $\bar{X}_A$ - Longitudinal location of special adaptor center of gravity, measured from faceplate mounting surface

   $\bar{X}_F$ - Longitudinal location of fixture center of gravity, measured from faceplate mounting surface

   $\bar{X}_s$ - Distance from CG of standard to mounting surface of pads

   $W_s$ - Weight of standard

2. **Definitions**

   Accuracy - A bias in the average of a set of readings from a standard.

   Balancing Moment - Restoring moment.

   Center of Rotation - When a warhead is mounted on the instrument and rotated in order to determine the radial location of the CG, the CG will describe a circle about the axis of rotation. (NOTE: The instrument faceplate and adaptor plate are designed to cause the center of rotation to fall on the warhead axis.)

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Precision - A measure of repeatability.

Restoring Moment - The moment required to re-establish a level balanced condition.

Sensitivity - The minimum moment required to produce a measurable imbalance per unit weight of warhead.

Warhead Axis - The line passing through the center of, and perpendicular to the plane of, the datum circle of the warhead. (On Littlejohn, this is on a tapered surface which is a portion of a right circular cone; on Honest John, it is a bolt circle.)

3. Conventions

Restoring radial moments are taken as positive when clockwise, viewed from rear.

The rear of the instrument is the end at which the levels are located.

# INTRODUCTION

Weight and center of gravity (CG) location are but two of many parameters which must be considered in the design of an ammunition item which, at some time during its brief functional life span, must conform to a desired trajectory, or path. With the advent of intercontinental ballistic missiles and their offspring, anti-missile missiles, the need for "bulls-eye" capability is particularly emphasized.

The design of equipment for measurement of center of gravity location for warheads has been left to the various contractors. It is not surprising, therefore, to learn that there exist as many different designs of CG equipment as there are contractors. The review of these designs and evaluation of the equipment performance has been, and continues to be, a major area of interest and activity of inspection engineering personnel of the Quality Assurance Division of Picatinny Arsenal.

Needless to say, considerable background has been obtained from this activity. Thus, in the design of the equipment described in this report, many of the desirable design features provided by other designers have been borrowed. However, in no case does there appear to be contractor data of tests to describe equipment performance to the extent reported herein. This is not said in condemnation; it must be recognized that the contractor is guided by considerations of time and economy. His CG equipment is, after all, but one or two of many measuring devices he must employ to sell his end item. That his CG equipment provides valid results has been the principal objective in the review of his design and his equipment. This can be done without the degree of testing reported here.

The Universal CG device described in these pages was designed on the basis of the following criteria desired to be inherent in its capabilities:

    a. Measure longitudinal and radial CG location by variables (rather than by attributes).

    b. Measure skin concentricity (this instrument does contain this capability, and the incorporation of this feature influenced the overall design. In particular, the weight of the instrument became heavier than desired, but not to the extent to jeopardize its CG measuring capabilities. The remainder of this report will confine itself solely to CG considerations).

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   c. Handle warheads in the weight range of approximately 200 to 2,000 pounds, and in the (longitudinal) CG range up to approximately 40 inches from a (rear) mounting plane.

   d. Be independent of the physical length of the warhead, provided the warhead meets the requirements of paragraph c.

   e. Provide an accuracy of at least $\pm$ 1/64 inch.

# DESCRIPTION OF INSTRUMENT

The design of this instrument employs the "restoring moment principle". The instrument and the warhead, when mated, becomes a single mass whose resultant center of gravity seeks its neutral, or equilibrium, position within the constraints imposed by the support. In this case, the support, or fulcrum, is (theoretically) a single point which permits three degrees of freedom: tilting in each of two perpendicular planes (Y-Z and X-Z) whose intersection is the plumb line through the point of support (Z-Axis), and rotation about this line. The latter motion is controlled and is not used in the operation of the equipment. See Figure 1.

I. Theory of Operating Principles

    A. Longitudinal CG

The basic instrument (see Photo 1) cannot balance by itself conveniently (or safely). In the photo, it is shown as being supported by contact at the fulcrum point (hidden by the bracket on the A-frame, said frame containing the cone supporting surface on the upper portion of the horizontal member; see Photos 4 and 7) and contact with the retractable jack, which rests on the floor. If the jack were to be lowered, the equipment would follow the jack downward because of the unbalanced moment, until its center of gravity coincided with the plumb line through the point of support. The situation is akin to an individual sitting on one end of a see-saw, alone (Figure 2a).

By adding a warhead, the "see-saw" tends toward a balanced condition (Figure 2b).

The final balance is achieved by adding known weights at a known (fixed) distance from the cone point, and the condition of balance (i.e., when the axis of the warhead is parallel to the ground) is evidenced by a precision level (Figure 2c).

It is to be noted that if the fixture and the warhead had tilted in the other direction, then weights would have been added at the front end of the horizontal fixture beam.

From the foregoing, the calculation for the longitudinal CG of the warhead $(\overline{X})$, is seen to be nothing more than a simple lever problem encountered in high school physics, or in statics. Figure 2c has been

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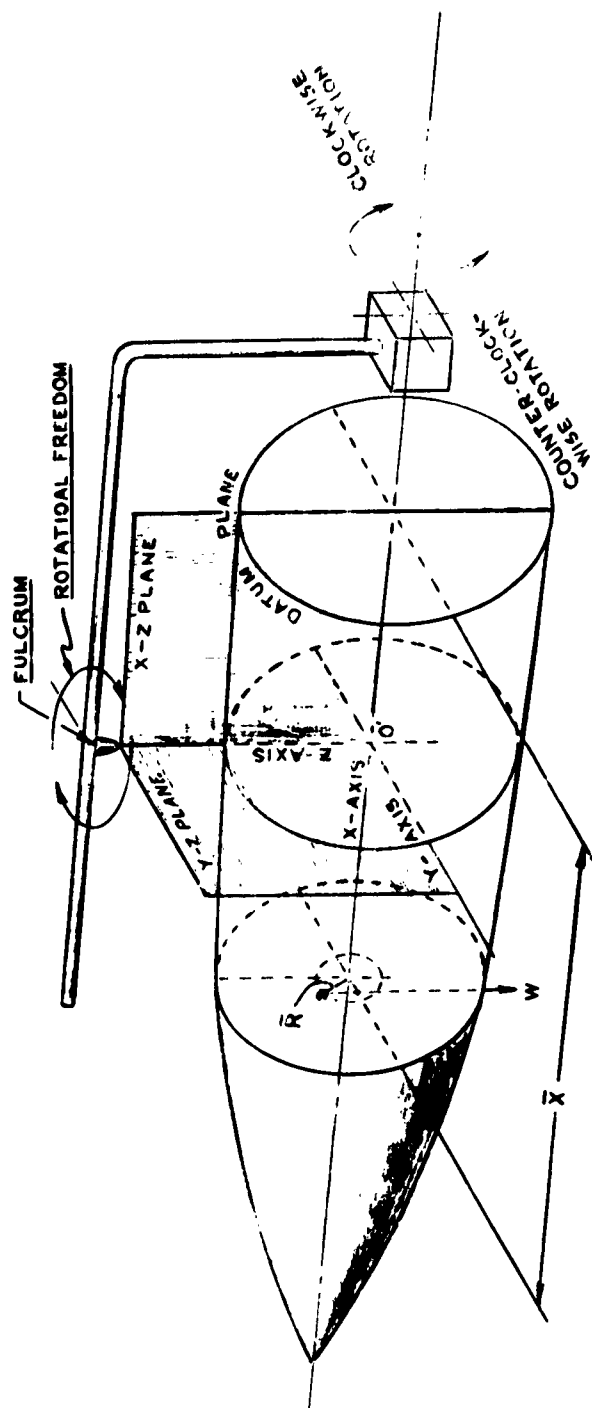
$\overline{X}$ = longitudinal CG from reference plane (measured along the warhead axis).

$\overline{R}$ = radial displacement of CG from warhead axis (measured perpendicularly to axis).

Figure 1

FRONT                                    REAR

FULCRUM POINT

FIXTURE WEIGHT

(a)

WARHEAD WEIGHT                    FIXTURE WEIGHT

(b)

64"

23.219

ADDED WEIGHTS

FACE PLATE

ONE LB.
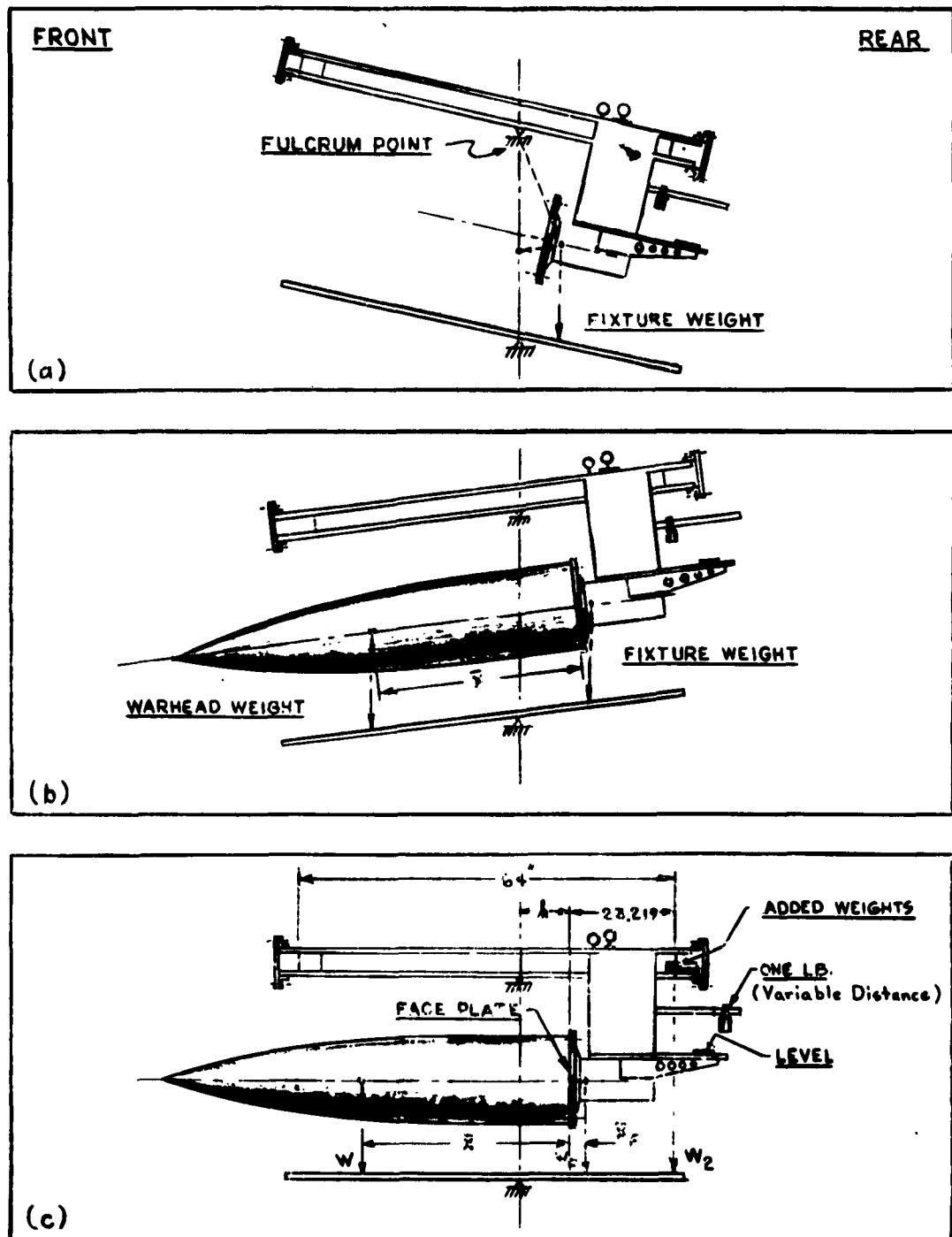(Variable Distance)

LEVEL

$W$        $\bar{x}$      $W_2$

(c)

Figure 2

embellished somewhat to show more of the quantities which must be considered. For a complete analysis of the calculation for $\overline{X}$, the reader is referred to Appendix A.

B. Radial CG

In the following discussion, the axis of the warhead is assumed to be collinear with the axis of rotation of the face plate. Furthermore, it is assumed that this axis and the pivot point determine a vertical plane. Three weights move in straight line paths perpendicular to this plane, and may be placed on either side of the plane, as required. Photo 5 shows the three radial weights in the neutral (zero) position.

The largest weight (ten pounds) can be positioned in slots at half-inch intervals up to 20 inches from the zero position; this permits radial moments to be set at five inch-pound increments up to 200 inch-pounds. A one pound weight can be positioned in slots at one inch intervals up to five inches from the zero position; this permits radial moments to be set at one inch-pound increments up to five inch-pounds. A 0.1 pound weight can slide continuously up to 12 inches to either side of zero, and its distance from zero can be read to the nearest 0.01 inch; this permits radial moments to be set at 0.001 inch-pound increments up to 1.2 inch-pounds. Lest the reader be deluded, it is pointed out, hastily, that the theoretical sensitivity of this instrument has been observed to be 0.1 inch-pound for the lighter (Littlejohn) warhead and 0.5 inch-pound for the heavier (Honest John) warhead. In other words, if sufficient care is taken, a displacement of the 0.1 pound weight of one inch will just produce a discernible movement of the level bubble when the Littlejohn warhead is attached. For the same effect to be observed when the Honest John warhead is attached, the 0.1 pound weight would have to be moved five inches. For a more complete discussion of the sensitivity of this instrument, see Appendix C.

Referring to Figure 3, assume that the warhead is in the general position with respect to its axis so that its centroid is at position $P_1$. The unbalanced radial moment is $Wr_1$, and requires a clockwise radial balancing moment of equal magnitude $(M_1)$. Now the warhead is rotated $90°$; either to position $P_2$ or to position $P_4$ (in this idealized situation, it is

immaterial, as will be shown shortly). Say the second position is $P_2$. Then a counter-clockwise radial balancing moment ($M_2$) is required, of magnitude $Wr_2$.

Thus we have two equations wherein the unknowns, $r_1$ and $r_2$ are evaluated:

$$Wr_1 = M_1 \quad , \quad r_1 = \frac{M_1}{W}$$

$$Wr_2 = M_2 \quad , \quad r_2 = \frac{M_2}{W}$$

Now the four right triangles, $P_1 O Y_1$, etc. are congruent (see Figure 3). Therefore, $Y_1 P_1 = O Y_2 = r_2 \ (= r_4)$ and $O Y_1 = Y_2 P_2 = r_1 \ (= r_3)$.

Consequently, $\bar{R}^2 = r_1^2 + r_2^2$

$$\text{or, } \bar{R} = \frac{\sqrt{(M_1)^2 + (M_2)^2}}{W}$$

Obviously, the following pairs of positions could be used in this case: $P_1$ & $P_2$, $P_2$ & $P_3$, $P_3$ & $P_4$, $P_4$ & $P_1$. With the presentation of this idealized concept, the reader is referred to Appendix B for a more general discussion of this matter, when the conditions assumed in the foregoing do not exist.

II. Design Features

   A. Fulcrum

The principle design feature of this instrument is the fulcrum point, which allows the centroid of the suspended mass to assume its equilibrium position along the plumb line through the (theoretical) point of support. Consequently, the device functions as a pendulum; more precisely, it may be described as a paraconic pendulum, because the oscillations are not confined to any one particular vertical plane.

This design of support was selected on the basis that the friction at the support would be of magnitude comparable to that of other support designs. Indeed, it was expected that frictional errors would be less than that created by other support designs (e.g., bearings, knife edges).

Another basis for selecting this type of support is that it provides the capability to measure the longitudinal and radial CG distances using one piece of equipment, without any additional operations other than the
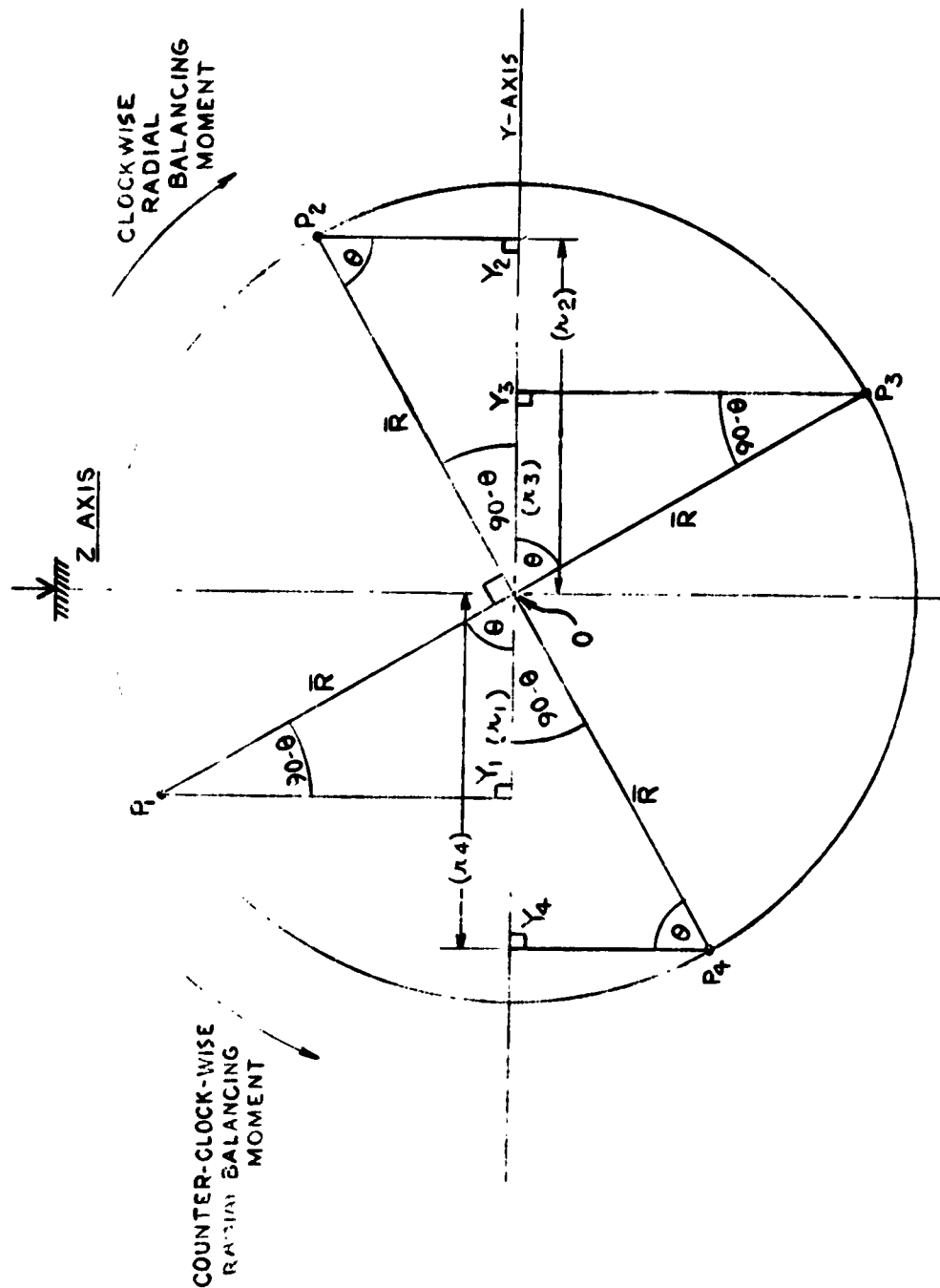
IDEALIZED GEOMETRY FOR C.G. MEASUREMENT (RADIAL)

Figure 3
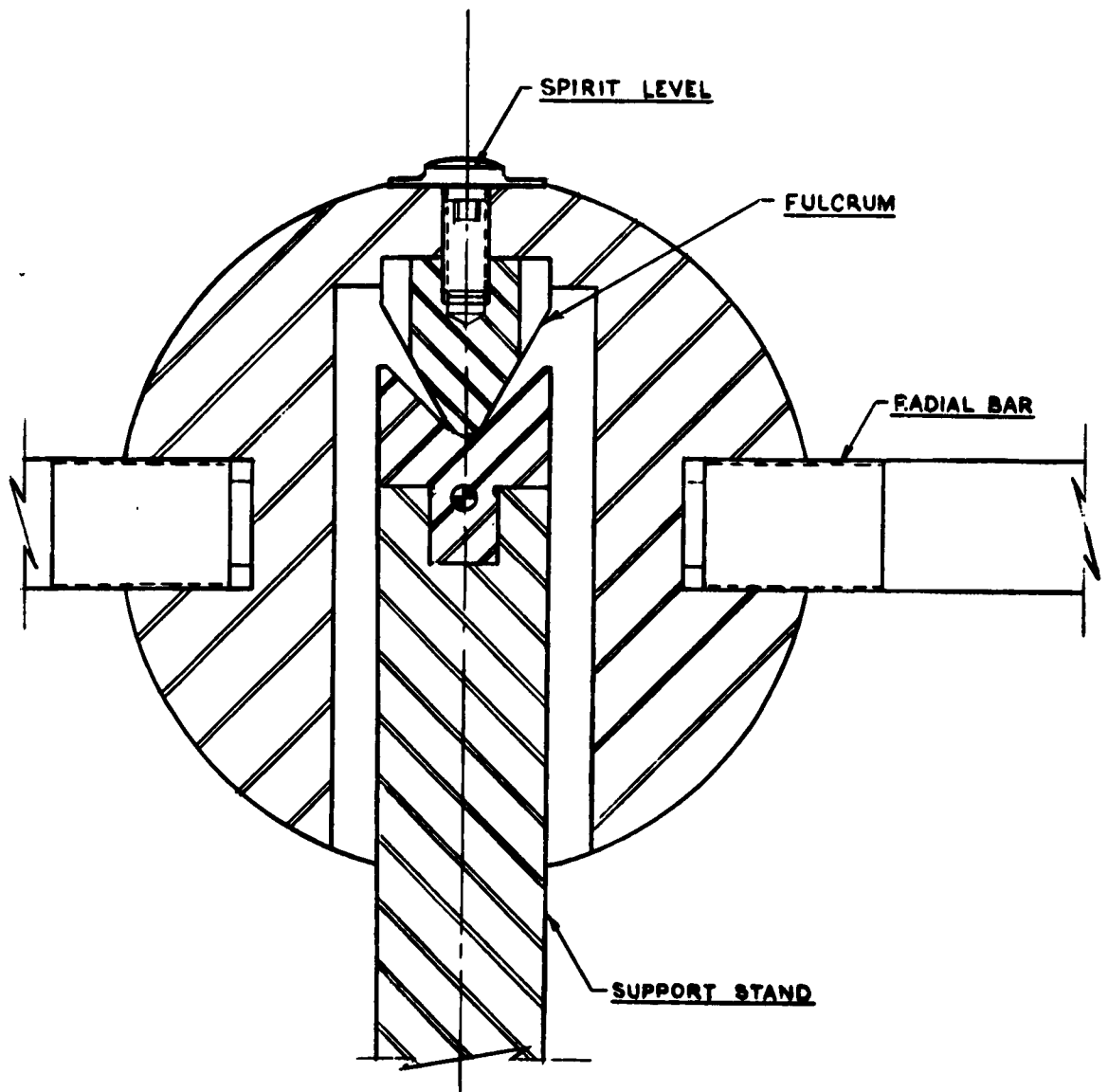
SPIRIT LEVEL

FULCRUM

RADIAL BAR

SUPPORT STAND

Figure 5

rotation of the warhead.

It is interesting to note that the original design of the fulcrum was a ball-and-socket affair. However, the slight discrepancies introduced by the ball "riding up" the socket were capable of being detected!. As a result, the socket was replaced by a plane (i.e., a socket of infinite spherical radius). Figure 4 illustrates the geometry of the error just discussed. For a more complete treatment of this matter, see Appendix C.

AXIS of SOCKET — AXIS of CONE

POINT of CONTACT
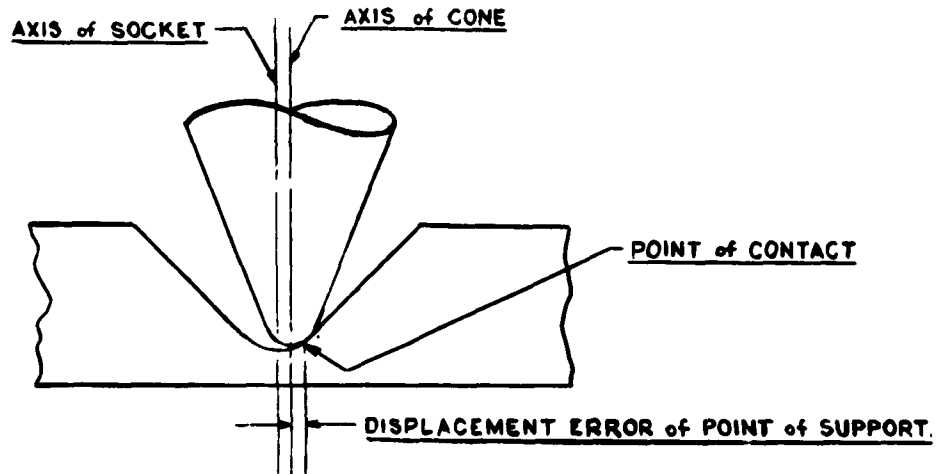
DISPLACEMENT ERROR of POINT of SUPPORT.

Figure 4.

It must be mentioned that brinelling of the cone support surface is to be expected. However, by suitable hardening of the surfaces, local strain deformations have been controlled to the extent that the operation of the equipment is not affected significantly. Theoretically, the cone point is deformed so that it is not spherical and, it is supposed, the support surface has a "socket" created by the cone point. Nevertheless, the data compiled to date indicates that the accuracy, precision, and sensitivity of the instrument are (compared to other similar devices) negligibly affected.

Finally, the matter of surface finish was considered so that frictional effects would be minimized.

B. Balancing Moments

A given moment may be achieved by an infinite number of combinations of force times lever distance. In the designing of this equipment, gross moments are obtained by the addition of various fixed weights placed at fixed lever distances.

a. Longitudinal: The horizontal I-beam of the device has been designed so that slotted weights can be located at either end, as required. These weights were designed (see Appendix D), so that the center of mass of each weight coincides with the bottom of the slot. The fit between the slot and the I-beam web prevents lateral movement, which would affect radial CG measurements.

b. Radial: The ten and one pound weights are capable of being located at discrete intervals, as discussed previously. However, these weights are always on the instrument and by virtue of their path, exert a constant longitudinal moment regardless of their lateral position.

In order to achieve final balance, longitudinally and radially, provision is made in each case for a sliding weight which can be positioned at the desired lever distance. The effective lever arm magnitude is read from a scale graduated in 0.01 inch. Originally, both weights were 0.1 pound; however, the longitudinal sliding weight is now 1 pound.

With the weights currently available, it may be necessary, in some cases, to create opposing moments to achieve longitudinal balance. By this, it is meant that weights may be added to both ends of the I-beam. The resultant balancing longitudinal moment is the algebraic summation of these moments.

C. Levels

Two levels are provided at the rear of the instrument, one each for longitudinal and radial inclinations. The movement of the bubble level of one marked division is equivalent to a vertical rise of 0.0005 inch per 12 inches, horizontally. Angularly, this corresponds to $0°0'9"$, or 0.00004 radians.

The selection of these levels is considered to be commensurate with the intended capability of this instrument, as evidenced by the data derived from the various tests, described elsewhere in this report.

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The use of less precise levels would be tantamount to taking a piece of equipment capable of weighing a horse fly and limiting it so that it could weight only a horse.

The plane upon which these levels are mounted is perpendicular to the face plate within $0^{\circ}0'20''$, and is parallel to the lines of travel of the radial weights within 0.001 inch throughout the entire range of traverse of these weights.

D.  Adjustability Features

In order to achieve a "universality" of application for measuring warhead CG, a "universality" of adjustment is capable of being achieved with the design of this instrument.

a.  The horizontal I-beam can be moved parallel to its length and set in a new position. At the present time, the I-beam is set as far forward as possible to offset the effect of the weight of the fixture to some extent (see Introduction).

b.  The position of the fulcrum point is adjustable along the length of the I-beam. The intended location (i.e., the k dimension) is such that, for minimum conditions (of weight and longitudinal CG, simultaneously), balancing would be achieved from the rear of the I-beam. However, as the weight of warhead increases, and if the longitudinal CG value, including its tolerance, is such that the requisite longitudinal restoring moment is incapable of being effected in the intended manner, it becomes necessary to calculate an optimum location for the fulcrum point so that balancing can be achieved by adding weights either at the front or the rear of the I-beam. A detailed analysis of this feature is contained in Appendix E.

c.  Adaptors:  The face plate which is integral with the instrument is designed for the Honest John warhead, which represents the "maximum condition" for the instrument. The front surface of this face plate establishes the reference plane for the warhead's longitudinal CG measurement. An adaptor is provided which attaches to the face plate, and establishes the reference plane for the Littlejohn, which represents the "minimum condition" for the instrument. Other adaptors to accommodate other warheads must be designed, as required, and corrections must be made to

reflect the effect of the added adaptor on the resultant CG location of the fixture. By following the method detailed in Appendix E, the equivalent of this effect is contained in the general equation to be found there.

E. "Universal" Standard

The standard is an "artificial" warhead whose weight and CG location are known prior to attaching it to the CG instrument for calibration purposes (see Photo 2). The calibration procedures are described elsewhere in this report.

In considering a design for a standard for a device which is intended for a gamut of warhead weights and CG distances, two possible concepts were realized:

a. Fixed Type: This is a standard which, by virtue of its symmetry of mass distribution, or simple geometry, permits a desired weight and CG location to be rather closely achieved. However, this requires a design with "tight" tolerances. For example, a simple illustration is a chunk of metal in the shape of a right circular cylinder of known length and diameter, with suitable provision for attaching it to the instrument. However this, and other simple geometries, were ruled out because of the inordinately large masses or enlarged dimensions that result from this approach. It must be borne in mind that the standard, in essence, provides a balancing moment, as does a normal warhead. If the geometry is to be "machineably" simple, then the weight factor and the CG location oppose each other. The example of the cylinder illustrates this lucidly. If the diameter is reduced to reduce the weight, the length must be increased to increase the CG distance, so that the required moment is capable of being exerted. Not only do awkward lengths result, but deflections in the standard due to its own weight increase to the point of introducing sizable errors. Conversely, reducing the moment arm by reducing the length requires an increase in the diameter (i.e., the weight) so that the desired moment is achieved. For these considerations, as well as the difficulties which would remain to determine the true CG location of the standard, this type of design was not used.

b. Adjustable Type: In considering the design difficulties discussed in the foregoing paragraph, the design concept of the existing

standard emerged (see Photo 2).

First, four pads with threaded holes were required to mate with screws located by the four holes in the large adaptor, or face plate. Next a pyramidal type framework of hollow, thick-walled pipe formed a rigid structure which located the four pads and allowed a concentration of mass toward the apex of the pyramid. Finally, a solid circular shaft of steel was added which, by suitable positioning, permitted longitudinal adjustment of the standard's CG so that it would coincide with its fulcrum.

The photograph shows the standard balanced on its own stand. The fulcrum, which cannot be seen in the photograph because it is hidden in a hole in the solid axial shaft, causes the standard, like the CG instrument, to behave as a paraconic pendulum. Although Figure 5 shows the original design as a ball-and-socket, the socket was replaced by a plane surface, for reasons discussed previously. The resulting precision in locating the standard CG was observed to be improved.

The two bars extending perpendicularly from the shaft permit a sliding weight to be positioned to achieve radial balance, during which process the longitudinal CG of the standard is not changed. Adjustment of the solid shaft in a direction parallel to its axis permits longitudinal CG to be varied as desired. A circular level indicates when longitudinal and radial balance exist simultaneously.

It is to be noted that the weight of the standard is constant although the CG location can be shifted. By suitable addition of longer solid shafts to replace the existing one, the weight of the standard can be increased, as well as the range of location of the longitudinal CG. In other words, the design permits a family of standards to be made relatively cheaply and easily, so that a desired standard moment may be selected for use.

At the present time, the standard exerts a restoring moment in the range required for the Littlejohn warhead. Consequently, relocation of the fulcrum for larger warheads is accomplished by computation and physical setting of the fulcrum to the calculated location. Ideally, the fulcrum location should be checked by the use of a standard which physically demonstrates the correctness of the fulcrum location.

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However, the point being made is simply the presentation of the idea that, lacking certain physical equipment, specifically, a standard for each fulcrum location, the shifting of a known starting point to a new starting point is capable of being accomplished by proper application of the principles involved. In the situation described here, it is reiterated that the principles of the lever laws are being employed to define the desired modification. If the data and the theory are valid, and are used correctly, and the modification is performed correctly, the resulting modified equipment is considered to be useful for its intended function.

F. Handling

1. Instrument: Two eyebolts are provided. In lifting the basic instrument alone, in a nearly balanced position, the rear eyebolt is used (see Photo 1). The longitudinal position of this eyebolt is adjustable; its nominal location was obtained by calculation of the instrument CG location prior to its fabrication.

The other eyebolt is integral with the fulcrum cage assembly and is used in the event it should be necessary to handle the instrument with the warhead attached.

2. Standard: The two eyebolts seen in Photo 2 are used with a bar and sling arrangement to lift and transport the standard to and from the instrument. They are removed prior to balancing and using the standard.
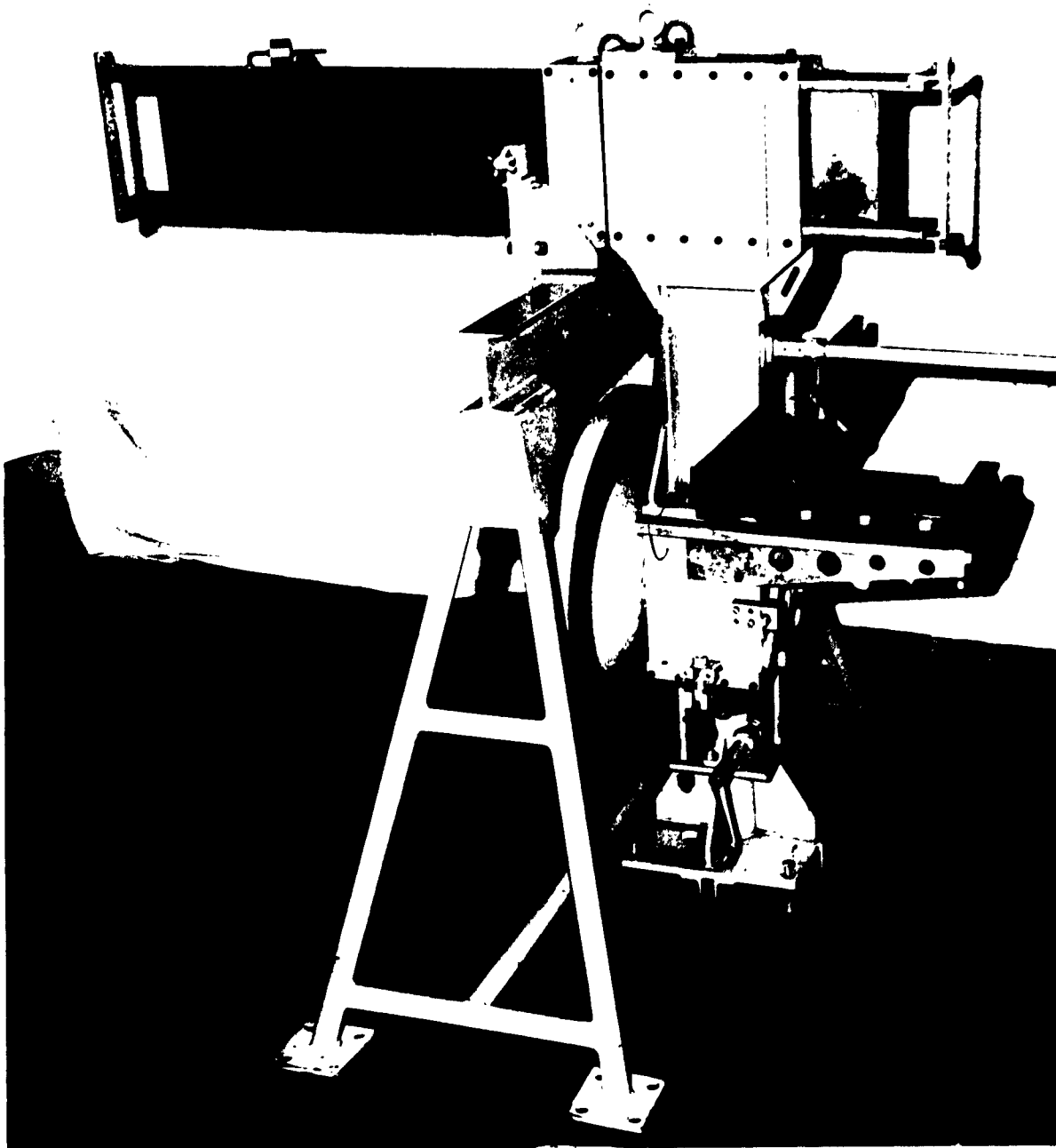
III. Outline of Procedures

The following outline of procedures is intended to describe the use of all of the equipment comprising the complete instrument. Bereft of all complicating details, the essence of the typical use of this equipment may be summarized: The basic principle of operation of the instrument is that which describes a simple lever whose fulcrum lies between the two loads which are applied to the lever. The moment exerted by the instrument is adjusted to achieve a balanced condition, so that the restoring moment (M) is known with accuracy. The unbalancing moment caused by the warhead is the product of a known weight (W) of warhead and an unknown lever distance ($\bar{X}$). Therefore, $W\bar{X} = M$ or $\bar{X} = \frac{M}{W}$, and the CG distance, $\bar{X}$, is seen to be a value derived from known quantities.

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However, in order to be able to achieve the situation whereby the instrument moment is indeed a known quantity, the standard first must be mated to the instrument; now the situation is reversed. The moment exerted by the standard ($M_s$) is known. The balancing moment (in its simplest sense) exerted by the instrument is the product of a known weight of fixture ($W_F$) and an unknown lever distance ($\overline{X}_F$). Therefore, $W_F\overline{X}_F = M_s$ or $\overline{X}_F = \dfrac{M_s}{W_F}$, and the CG of the fixture, $\overline{X}_F$, is seen to be a value derived from known quantities (see Appendix F).

## OUTLINE OF PROCEDURES

| OPERATION | PARAMETER OBTAINED | REFERENCE | REMARKS |
|---|---|---|---|
| **A. Calibration** | | | |
| 1. Weigh & balance standard | $W_s, \bar{X}_s$ | pp 19, 20, F-1 | |
| 2. Weigh fixture (less pivot cage) | $W_F$ | p A-1(2c) | |
| 3. Establish fixture CG | $\bar{X}_F$ | pp A-1(2d), F-1 | Fixture is balanced longitudinally and radially, using standard. Radial CG of fixture is made = 0 |
| 4. Establish pivot location | k | pp A-2, E-2 | This will be minimum k for present fixture design |
| **B. $\bar{X}$ Measurement** | | | |
| 1. Assemble adaptor to faceplate, as required | k | | If required, $\bar{X}_F$ will shift. Effect of adaptor must be considered. |
| 2. Establish new k value, if necessary, and adjust pivot cage accordingly | k | pp E-1 to E-7 | Calculated k value may have to be re-calculated on basis of optimum location, depending on warhead weight and CG requirements. |
| 3. Assemble warhead to adaptor, balance and calculate $\bar{X}$ | $\bar{X}$ | pp E-7 to E-14 | One value to be used for radial CG calculation is also obtained at this time. |
| **C. $\bar{R}$ Measurement** | | | |
| 1. Continuing from B3, rotate warhead to each of three positions, 90° apart, balancing at each position, and calculate $\bar{R}$ | $\bar{R}$ | pp B-1 to B-8 | Requires four positions, in general. |

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INDEX TO PHOTOGRAPHS

Photograph No. 1

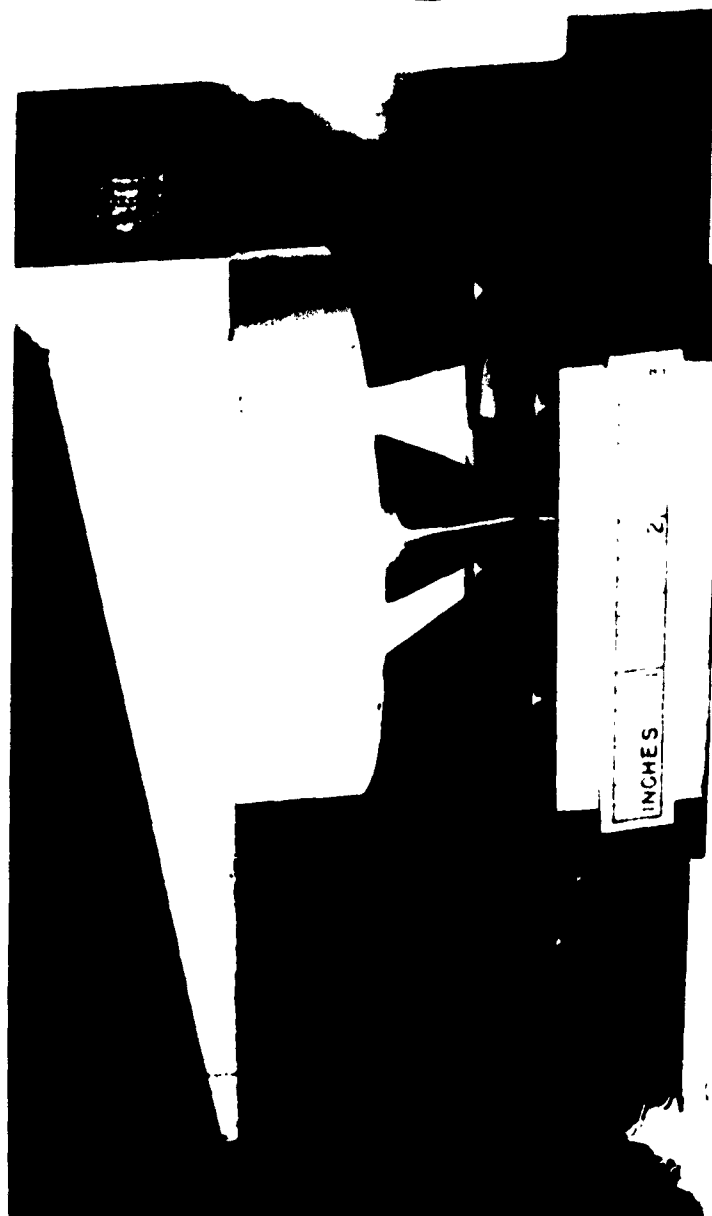Instrument Resting on Fulcrum and Jack

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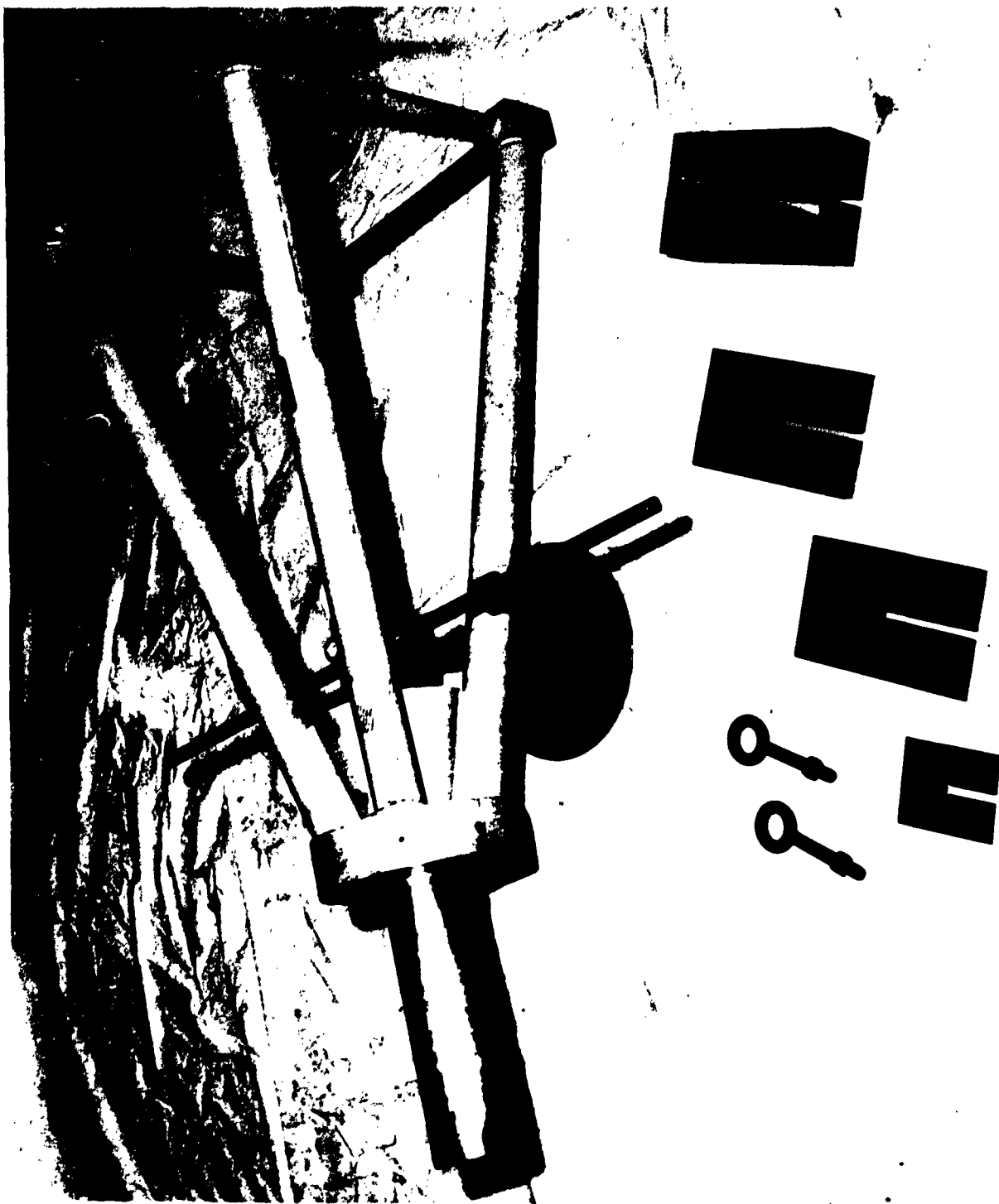
Photograph No. 2

Standard and Weights
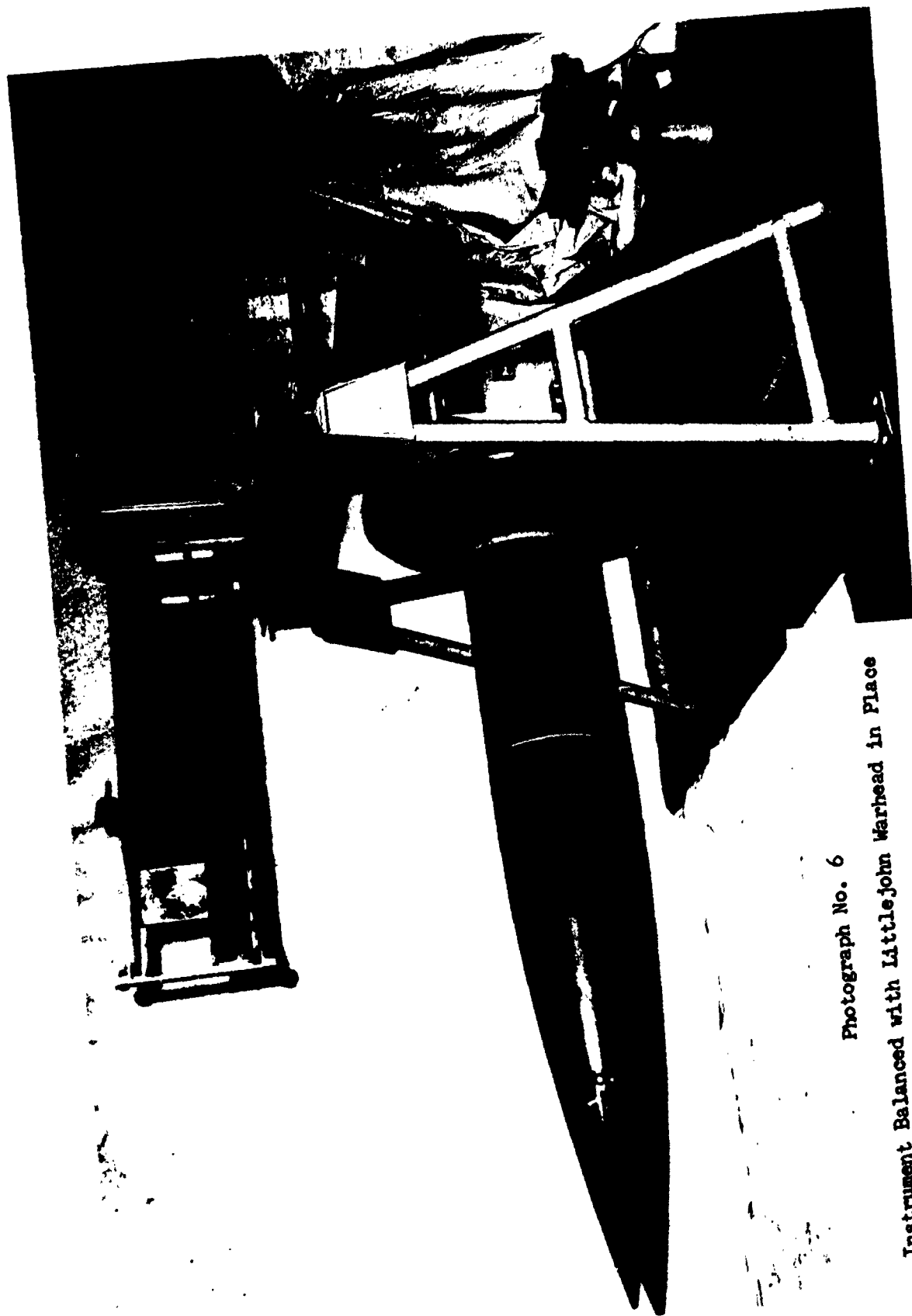
Photograph No. 3

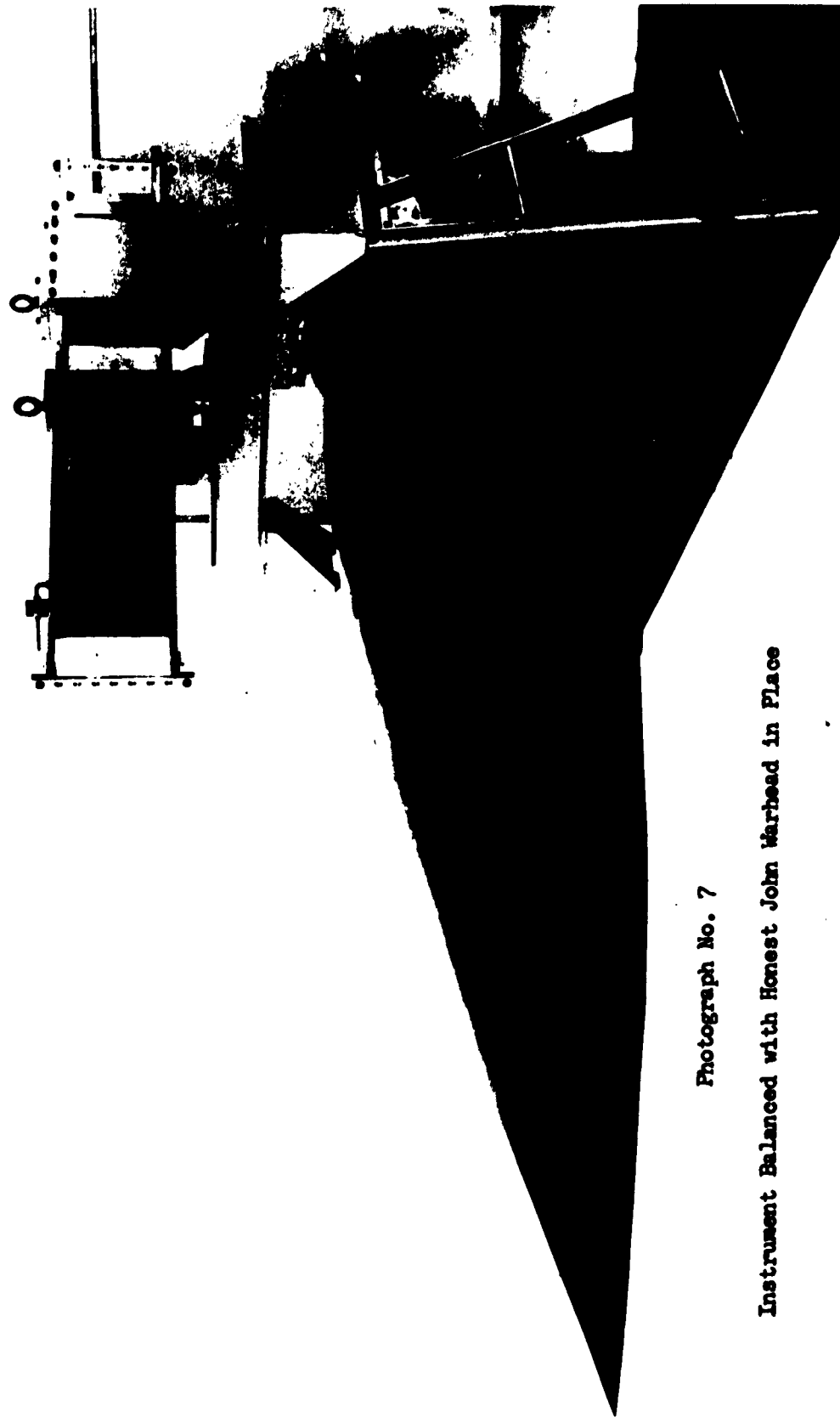Instrument Balanced on Fulcrum, Standard in Place, Jack Removed

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Photograph No. 4

Close-Up of Fulcrum

Radial Weights - 10 lb.

1 lb.

0.1 lb.

Photograph No. 5

Rear of Instrument

28

Photograph No. 6

Instrument Balanced with LittleJohn Warhead in Place

29

Photograph No. 7

Instrument Balanced with Honest John Warhead in Place

30

## INDEX TO APPENDICES

31

# APPENDIX A

1. Derivation of general equation for longitudinal location of warhead center of gravity.
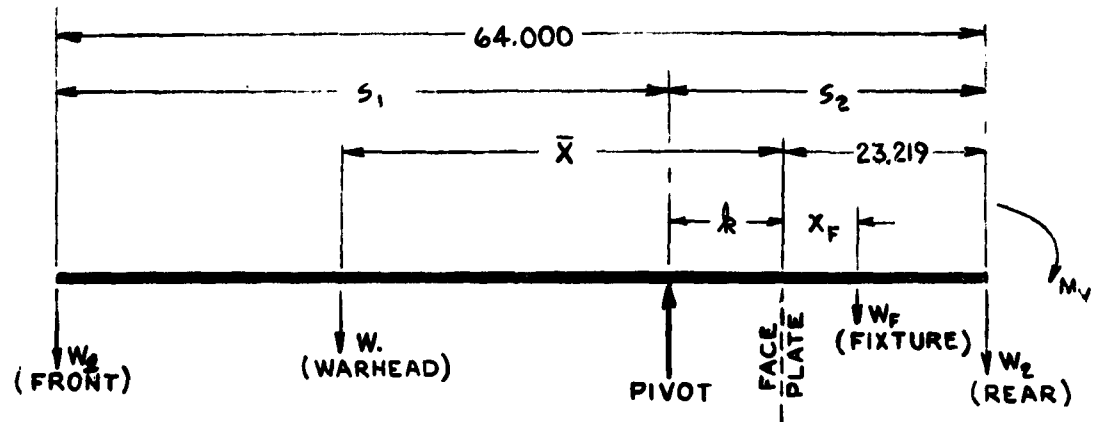


Figure A-1

For equilibrium,

$$\Sigma M_{pivot} = W_1(40.781-k) + W(\bar{X}-k) - W_F(\bar{X}_F+k) - W_2(23.219+k) - M_v = 0$$

Let $M_1 = W_1(40.781-k)$

$M_2 = W_2(23.219+k)$

$$\bar{X} = \frac{W_F(\bar{X}_F+k) + M_2 + M_v - M_1}{W} + k$$

Let $M_2+M_v-M_1 = $ net moment $= M$

$$\bar{X} = \frac{W_F(\bar{X}_F+k) + M}{W} + k$$

2. Basic constants

   a. Fixture total weight = 1729 lbs.

   b. Pivot cage weight = 36 lbs.

   c. $W_F$ (net fixture weight) = 1693 lbs.

   d. $\bar{X}_F$ (distance, face plate to fixture CG) = 4.389"

   e. $W_1$ CG to $W_2$ CG = 64.000" ($= s_1+s_2$).

   NOTE: All values used refer to the initial position of the beam in the main cage. If this is changed, new values for $\bar{X}_F$, $s_1$, $s_2$, & k will have to be determined.

A-1

## APPENDIX A

### INSTRUMENT CONSTANTS

### 3. Tabulation of Data for Various Warheads

| Warhead | k | $s_1$ | $s_2$ | $\bar{X} = \dfrac{W_F(\bar{X}_F+k)+M}{W} + k$ | D* |
|---|---|---|---|---|---|
| T-54, M146, M5 | .798 | 39.983 | 24.017 | $\dfrac{8812.461+M}{W} - .466$ | .018 |
| Littlejohn | | | | (Special case, including effect of adaptor plate and giving $\bar{X}$ as defined in drawing 8849040) | |
| | .798 | 39.983 | 24.017 | $\dfrac{8781.591+M}{W} + .798$ | .018 |
| | | | | (For warhead mounted directly against faceplate) | |
| M1A2 HonestJohn | 10.262 | 30.519 | 33.481 | $\dfrac{24804.143+M}{W} + 10.262$ | 9.482 |
| M38 M144 M132 HonestJohn | 14.712 | 26.069 | 37.931 | $\dfrac{32337.993+M}{W} + 14.712$ | 13.932 |
| XM46 HonestJohn | 14.916 | 25.865 | 38.135 | $\dfrac{32683.366+M}{W} + 14.916$ | 14.136 |

A-2

*D = Measured distance between pivot cage and main support cage, required to establish a given value of k; it is measured at left bottom edge, looking from rear. (D = k - .780) See Figure E1

PROCEDURE FOR DETERMINING RADIAL LOCATION

of

T54, M146 or M8 LITTLEJOHN CENTER OF GRAVITY

1. Balance radially, and read balancing moments, at each of four positions, A, B, C, D of the faceplate (clockwise balancing moments are considered positive, counterclockwise negative).

2. Calculate $M_1 = \dfrac{M_A - M_C}{2}$ and $M_2 = \dfrac{M_B - M_D}{2}$ . Record, disregarding sign.

3. Using Chart BI, place arrow of "resultant" strip on value of $M_1$, align edge of strip at $M_2$. Read M, Resultant Moment, where strip edge intersects the $M_2$ line.

4. Any value of M below 25.3 inch-lbs, indicates an acceptable warhead. For higher values see Chart BII. If the intersection of M and warhead weight falls below the lower line, the warhead is acceptable. Any such intersection points above the lower line represent reject warheads; if they fall between the two lines, consult Engineering for re-check before rejecting.

If it is desired to know the actual radial displacement for any given warhead, refer to Chart BIII.

See Appendix B-2 for an example of this procedure.

EXAMPLE OF DETERMINATION OF RADIAL LOCATION OF

T54 LITTLEJOHN CENTER OF GRAVITY

W = 258 lbs.

1. Moment readings:

| | A | B | C | D |
|---|---|---|---|---|
| | +25 | -22 | -5 | +18 |

2. $M_1 = \dfrac{25-(-5)}{2} = 15$

$M_2 = \dfrac{-22-18}{2} = -20$

Record as $M_1$ = 15, $M_2$ = 20

3. M = 25

4. Since M = 25 is below 25.3 inch-lbs., the measured warhead is well within tolerance. The actual displacement, shown on Chart BIII is .097". Assume, now, that M had been 25.9. Reference to Chart BII and travelling vertically along the 258 lb. warhead weight line, it is seen that M = 25.9 represents a reject. If M had fallen between 25.3 and 25.8, it would be in the range considered dangerously close to the acceptance limit of $\bar{R}$ = .100", and the engineer in charge should personally investigate to determine acceptability of the item.
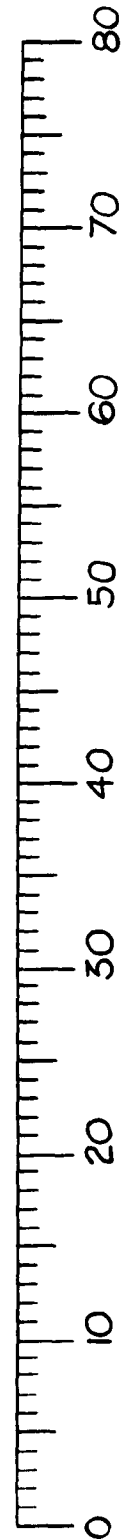
CHART B-I

RADIAL ECCENTRICITY

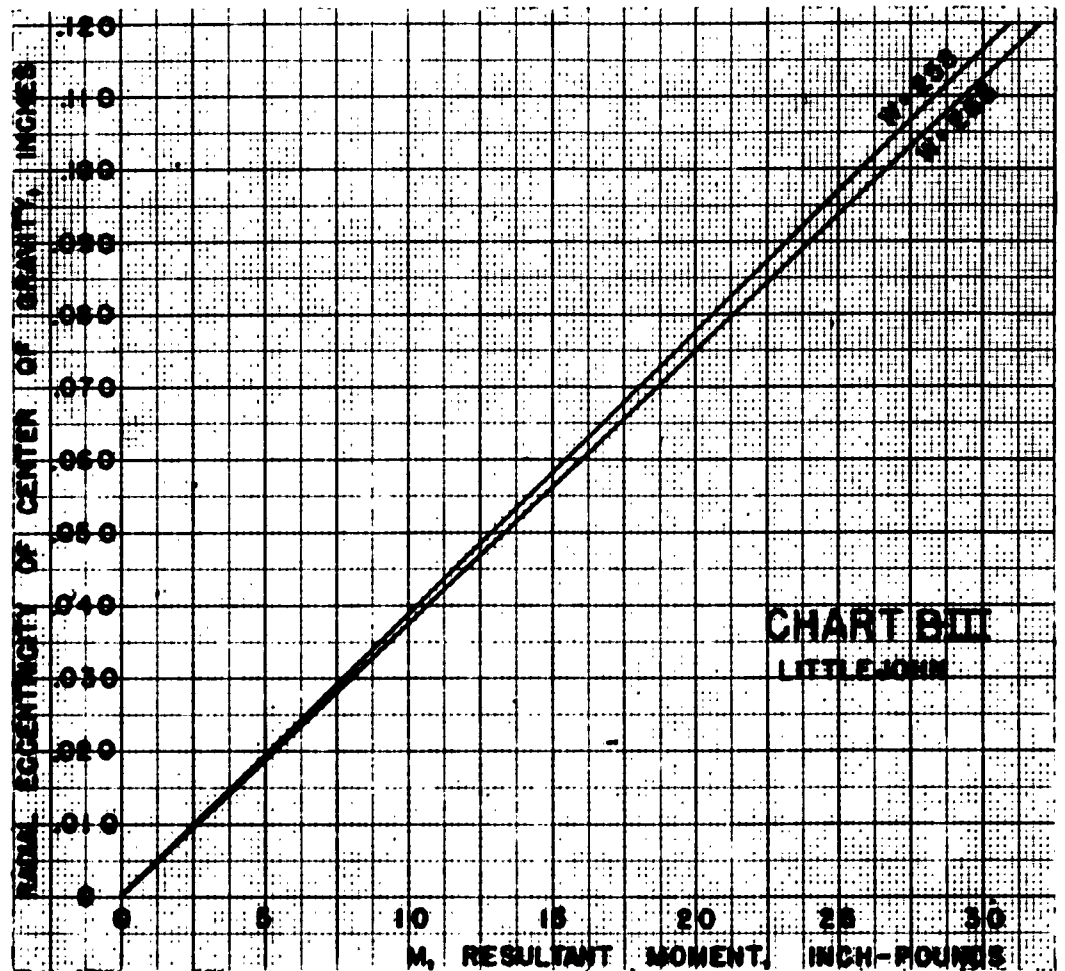DETERMINATION OF RESULTANT MOMENT

RESULTANT STRIP POSITION
IN EXAMPLE

M. INCH-POUNDS

M₂, INCH-POUNDS
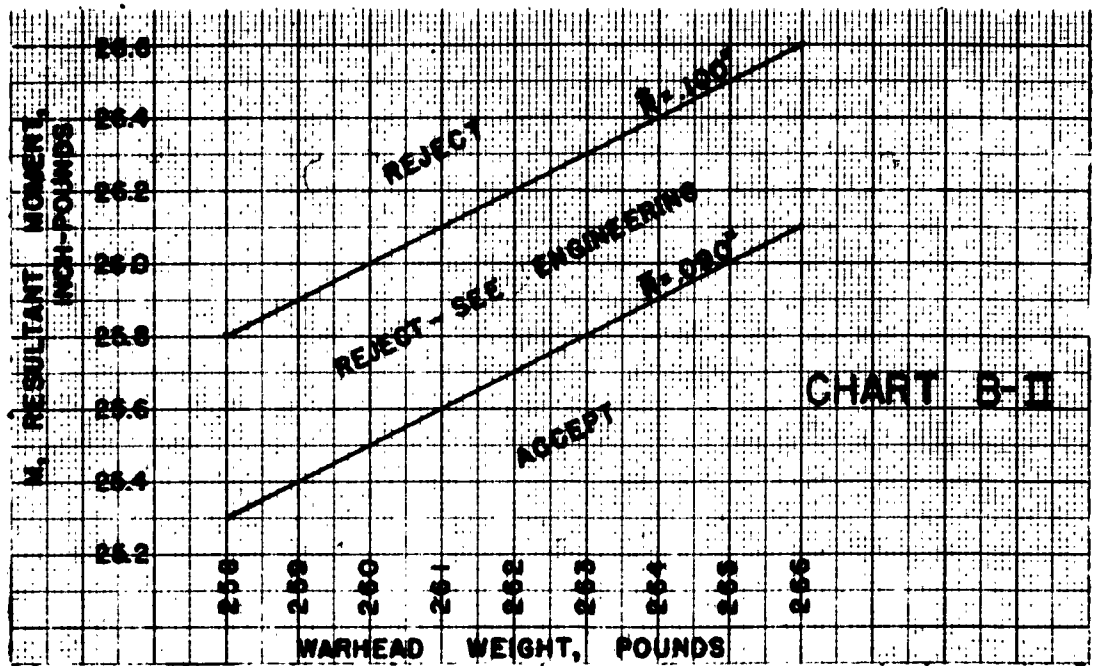
$M_2$, INCH-POUNDS

B-3
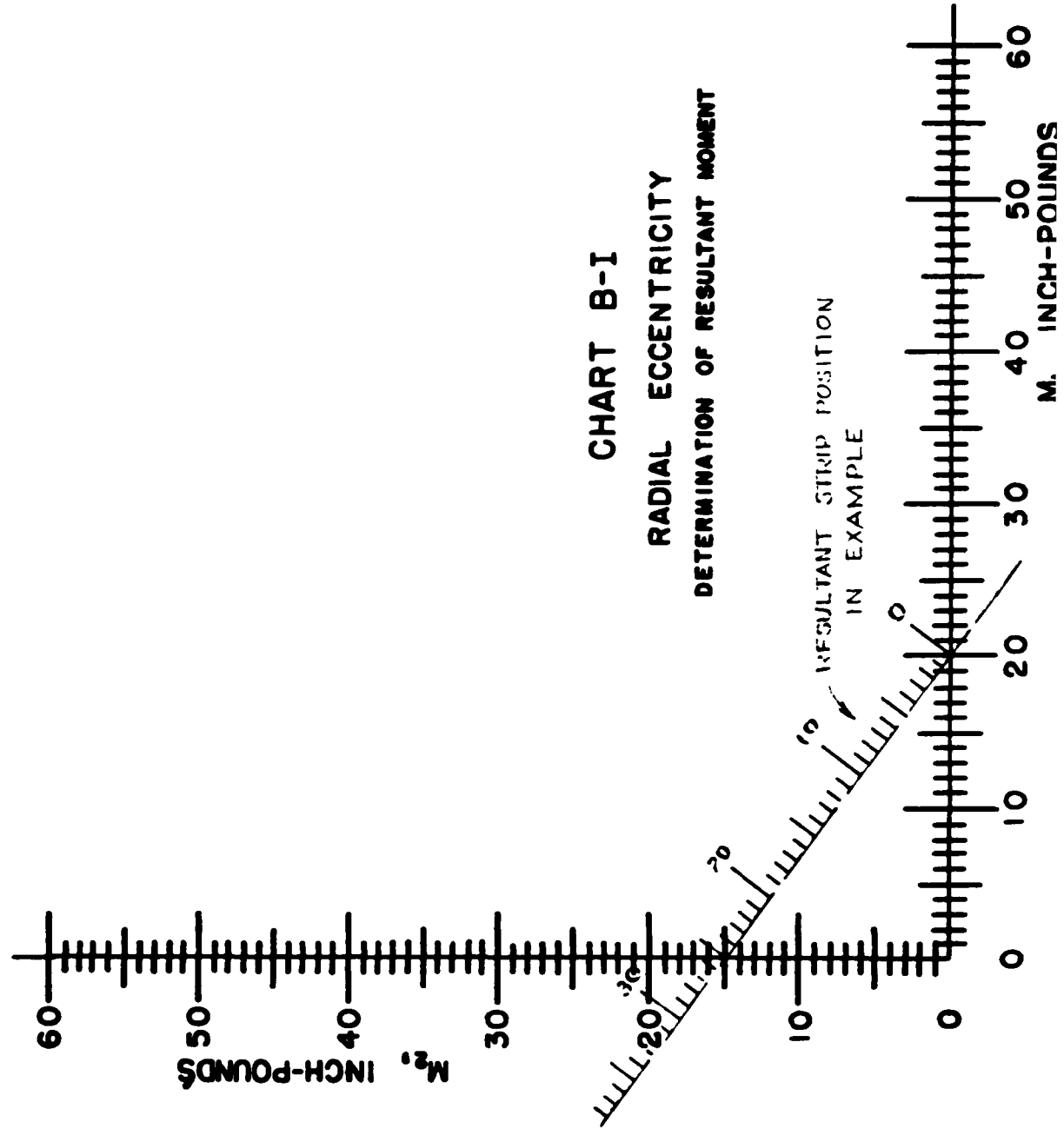
# APPENDIX B
## RESULTANT MOMENT STRIP
## FOR USE WITH CHART B-I

Directions:

Remove strip to right, or use ruler graduated in inches and tenths of inches, where $\frac{1}{10}$ inch will equal one inch-pound and 1 inch will equal 10 inch-pounds.

CHART B-II

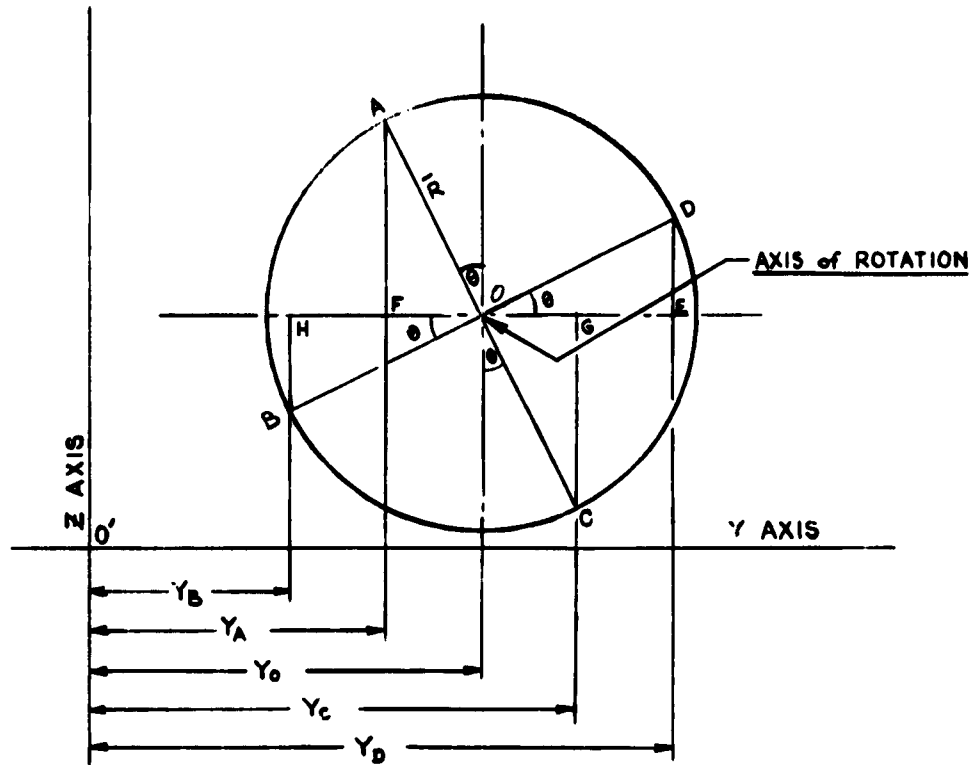M, RESULTANT MOMENT, INCH-POUNDS

REJECT

REJECT - SEE ENGINEERING

R=.100"

R=.090"

ACCEPT

WARHEAD WEIGHT, POUNDS



CHART B-III

LITTLEJOHN

RADIAL ECCENTRICITY OF CENTER OF GRAVITY, INCHES

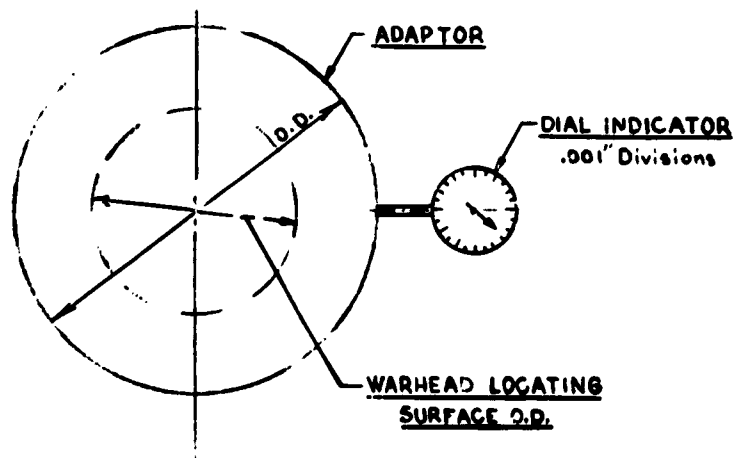M, RESULTANT MOMENT, INCH-POUNDS

Figure  B-1

Figure  B-2

B-6

## APPENDIX B-3
### DERIVATION OF PROCEDURE FOR DETERMINING
### RADIAL LOCATION OF WARHEAD CENTER OF GRAVITY

Any displacement of the center of rotation of the warhead center of gravity (warhead axis) from a vertical plane through the pivot point will result in radial moment readings, taken $180°$ apart, not symmetrically disposed about zero.

Refer to Figure B1. The z-axis is taken as the vertical through the pivot point, and the x-axis is the theoretical zero axis of the instrument; that is, if the axis of rotation of the warhead coincided exactly with the x-axis, then $y_B$ and $y_D$ would be equal in magnitude and opposite in sign, and similarly for $y_A$ and $y_C$.

It may be assumed that the center of the faceplate - the axis of rotation - is not precisely below the pivot point when the instrument is level radially. Thus, when a warhead is mounted concentric with the faceplate, a constant moment will be required to level the instrument, and the effect of radial displacement $(\overline{R})$ of warhead center of gravity from the warhead axis is superimposed on this constant value. As the warhead is rotated to four positions successively separated by $90°$, and balance is established at each position, readings of restoring moments will be obtained for positions at A, B, C, and D. When the measured moments are divided by the warhead weight, actual radial displacement of the CG is obtained, as follows:

| Warhead Position | Moment | $y = \frac{M}{W}$ |
|---|---|---|
| A | $M_A$ | $y_A$ |
| B | $M_B$ | $y_B$ |
| C | $M_B$ | $y_B$ |
| D | $M_D$ | $y_D$ |

Referring to Figure B1,

$$\overline{OF} = \overline{OG} = \overline{HB} = \overline{ED} = \frac{y_C - y_A}{2}$$

$$\overline{OE} = \overline{OH} = \overline{GC} = \overline{FA} = \frac{y_B - y_D}{2}$$

$$R = \sqrt{OF^2 + FA^2}$$

The displacement of warhead axis $= y_o$, and

$$y_o = \frac{y_A + y_C}{2} \qquad \text{or} \qquad y_o = \frac{y_B + y_D}{2}$$

Experimental data has established a maximum $y_o = .030"$.

These operations may be reduced to graphical form, as shown in the "Procedure for Determining Radial Location of Littlejohn Center of Gravity".

When an adaptor is added to the fixture to accomodate a given warhead, it may, in general, be accentric to the center of rotation. The effect of this eccentricity can be determined as follows:

1. Measure mechanical eccentricity of adaptor O.D. by use of a dial indicator placed to read at the right end (looking from rear) of a horizontal diameter (Figure B2). (Indicator support may be clamped to the I-beam). Set indicator at zero when faceplate is at A. Record values when faceplate is at positions B, C, and D, as b, c, and d. Pay strict attention to sign of reading.

2. Measure mechanical eccentricity of warhead locating surface O.D. similarly. Record as $b^1$, $c^1$, $d^1$.

3. Compute moment errors as $E_1 = (\frac{c}{2})W_A$

$$E_2 = (\frac{b-d}{2})W_A$$

4. Compute mechanical shift errors as $E_3 = \frac{c^1}{2}$

$$E_4 = \frac{b^1 - d^1}{2}$$

5. Take moment readings at positions A, B, C, D with warhead in place. Record as $M_A$, $M_B$, $M_C$, $M_D$, conforming to established sign convention.

6. Correct measured moments as follows:

$M_A - E_1$

$M_B + E_2$

$M_C + E_1$

$M_D - E_2$

B-8

7. Calculate $y_A$, $y_B$, $y_C$, $y_D$ by dividing the corrected moments by W.

8. Correct $y_A$, $y_B$, $y_C$ and $y_D$ as follows:

$$Y_A = y_A - E_3$$

$$Y_B = y_B + E_4$$

$$Y_C = y_C + E_3$$

$$Y_D = y_D - E_4$$

9. Then $\bar{R} = \sqrt{(\dfrac{Y_A - Y_C}{2})^2 + (\dfrac{Y_B - Y_D}{2})^2}$
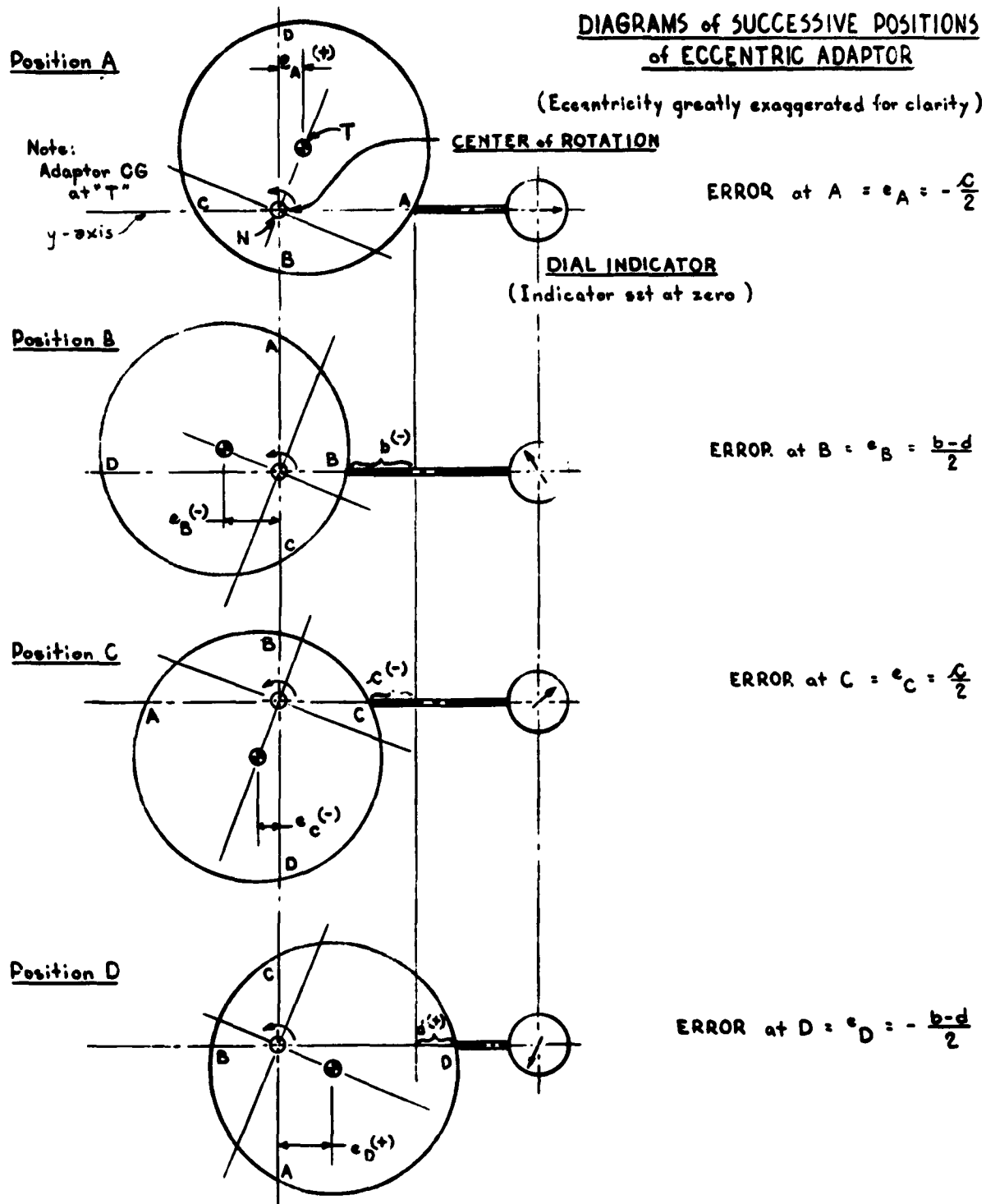
( Eccentricity greatly exaggerated for clarity )

**Position A**

$e_A (+)$

T

Note:
Adaptor CG
at "T"

CENTER of ROTATION

C

A

y-axis

N

B

ERROR at A $= e_A = -\dfrac{c}{2}$

DIAL INDICATOR

( Indicator set at zero )

**Position B**

A

D

B

$b (-)$

$e_B (-)$

C

ERROR at B $= e_B = \dfrac{b-d}{2}$

**Position C**

B

A

C

$c (-)$

$e_C (-)$

D

ERROR at C $= e_C = \dfrac{c}{2}$

**Position D**

C

B

D

$d (+)$

$e_D (+)$

A

ERROR at D $= e_D = -\dfrac{b-d}{2}$

Figure B-3

APPENDIX B-4

ANALYSIS OF EFFECT OF ECCENTRICITIES IN ADAPTORS

Refer to Figure B3

$\overline{NT}$ is the actual eccentricity of the adaptor O.D. After setting indicator at zero when faceplate is in position A, readings b, c, and d will be obtained. It is evident that the y-component of this eccentricity with respect to the center of rotation will appear as $e_C = \frac{c}{2}$ at position C, $e_B = \frac{b-d}{2}$ at position B, $e_A = -\frac{c}{2}$ at position A, and $e_D = -\frac{b-d}{2}$ at position D.

At position A in example, the centroid of the adaptor is shown with a y-component displaced $e_A$ from the center of rotation. This will produce an error of moment $W_A e_A$, shown here as producing a clockwise or positive moment which requires a negative restoring moment in the instrument. Therefore, in order to correct the result, a positive correction factor must be applied. Following the definitions of $E_1$, $E_2$, $E_3$ and $E_4$ given, the correct sign conventions to apply to the corrections are given in the preceeding tables. A similar analysis applies to any error of eccentricity in the warhead locating surface.

## APPENDIX C
## PRECISION, ACCURACY AND SENSITIVITY

Initial testing for instrument repeatability with the standard in place indicated a mechanical hysteresis, typified by results of a radial balance test shown in Graph C-I. Investigation determined that the pivot socket was softer (Rockwell C-38) than specified (R-C-63 to 65). The soft steel brinelled under load, and the spherical pivot climbed up the sides of the indentation, shifting the actual contact point away from true center and giving rise to errors of about 4½ inch-lbs. (Note that the curved portions at each end of the graph are caused by the bubble impinging the end of the vial, thus giving an apparently non-linear relationship, since the position of the bubble end was taken as giving a true angular reading while in reality it did not).

When a flat plate of hardness Rockwell C-64 was substituted, the total error was reduced to 2 inch-lbs. (Graph C-II).

A check of repeatability within small angles of tilt yielded Graph C-III which was analyzed by least-square curve fitting methods and statistical procedures. The response of the instrument was determined by this means to be 6.1 seconds of angular deviation per inch-pound of offsetting moment. The standard error of estimate ($S_y$) was established as 0.32 inch-lbs. A range of $\pm 2$ $S_y$ includes 95% of all readings; therefore, 95% of the time, any reading will fall within $\pm.64$ inch-lb. This compares favorably with the value established by general observation following.

When an experienced operator balances the instrument, observing maximum care, the minimum offsetting moment which can be detected by noting level bubble movement is the maximum sensitivity of the instrument. Dividing this minimum moment (inch-lb) by the weight of the warhead involved translates sensitivity in inch-lbs to sensitivity in terms of the CG location in inches. The following table shows observed and calculated values obtained in this manner:

|  | Moment, observed | $\bar{X} = \frac{M}{W}$ |
|---|---|---|
| 1. With Standard (324 lbs) | $\pm.1$ in-lb | |
| With Littlejohn (262 lbs) | | $\pm.0003"$ |
| 2. With Honest John (1687 lbs) | $\pm.5$ in-lb | $\pm.0003"$ |

It is evident that the sensitivity of the instrument, within the range 260-1690 lbs is constant in terms of CG location.

A more practical statement of sensitivity, reflecting values which might normally be expected in actual use, is based on the ability to read the level to $\pm\frac{1}{4}$ division with no special care. This is in accordance with standard estimates of instrument reading uncertainty, and also eliminates concern for minor variations such as might be introduced by temperature changes during a set of readings. One division is equivalent to 9 seconds of angle; as noted above, one inch-pound has been found to produce a tilt of 6.1 seconds with the standard (weight = 324 lbs); thus usable sensitivity may be taken as

$$\overline{X} = \frac{1\text{in-lb}}{6.1\text{sec}} \times \frac{9\text{sec}}{\text{div}} \times \frac{1\text{div}}{2} \times \frac{1}{324\text{lb}} = \frac{.75\text{in-lb}}{324 \text{ lb}} = .0023"$$
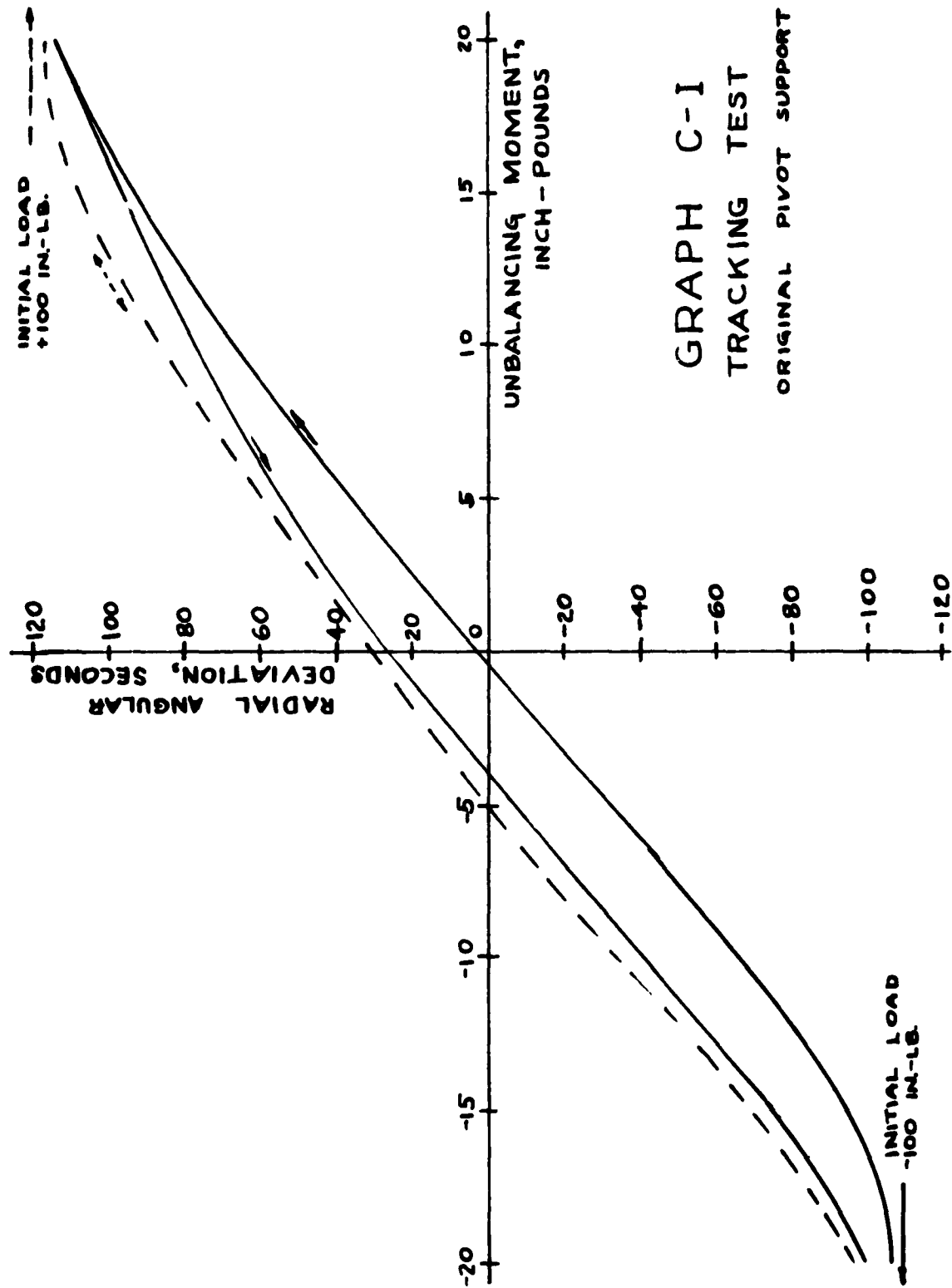
Since we have determined that sensitivity in determination of CG location is constant, the .0023" figure will apply as well to the Honest John warhead, or any other within the usable range of the instrument.
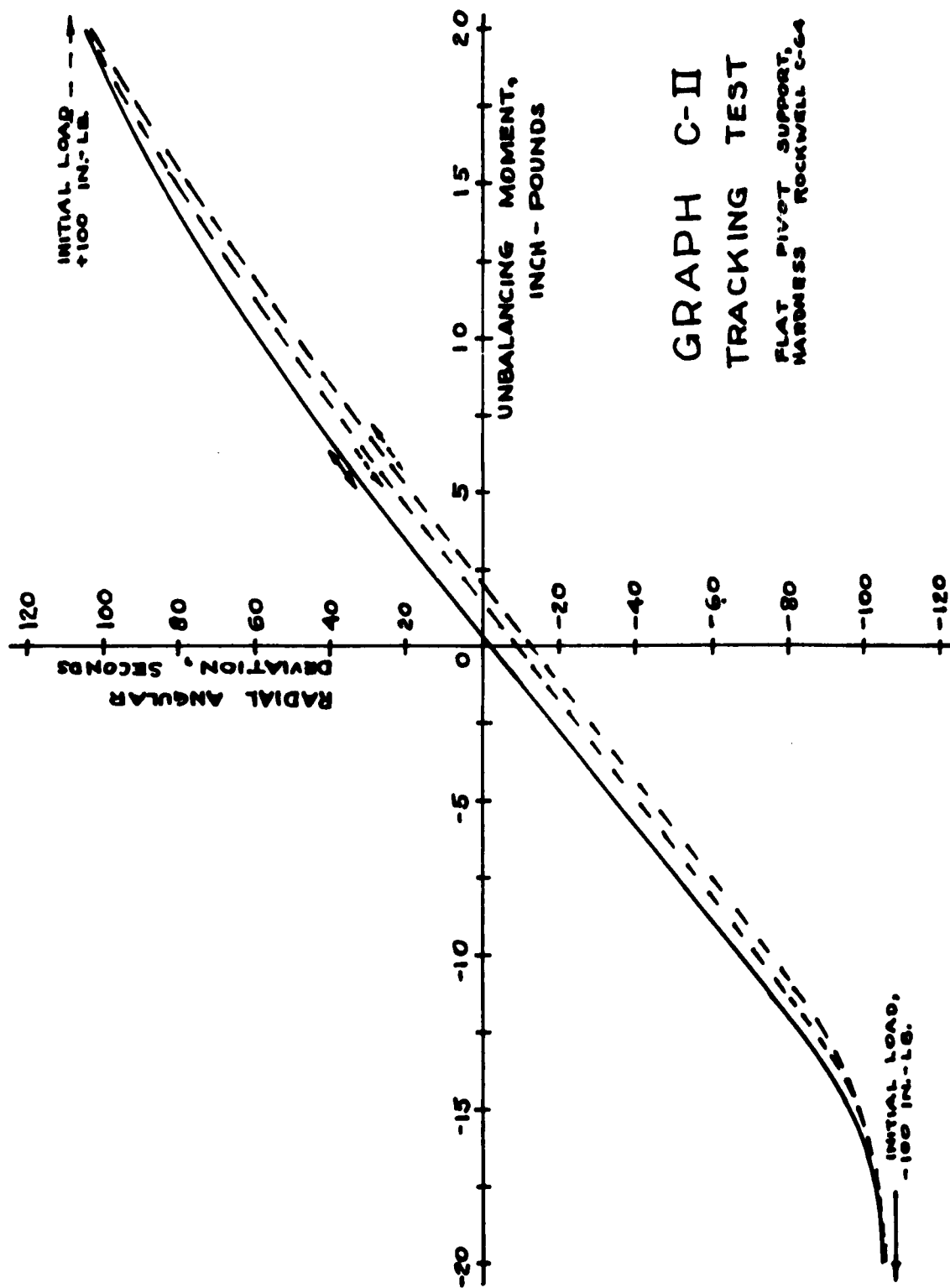
The present limits of acceptance for two warheads under consideration are:
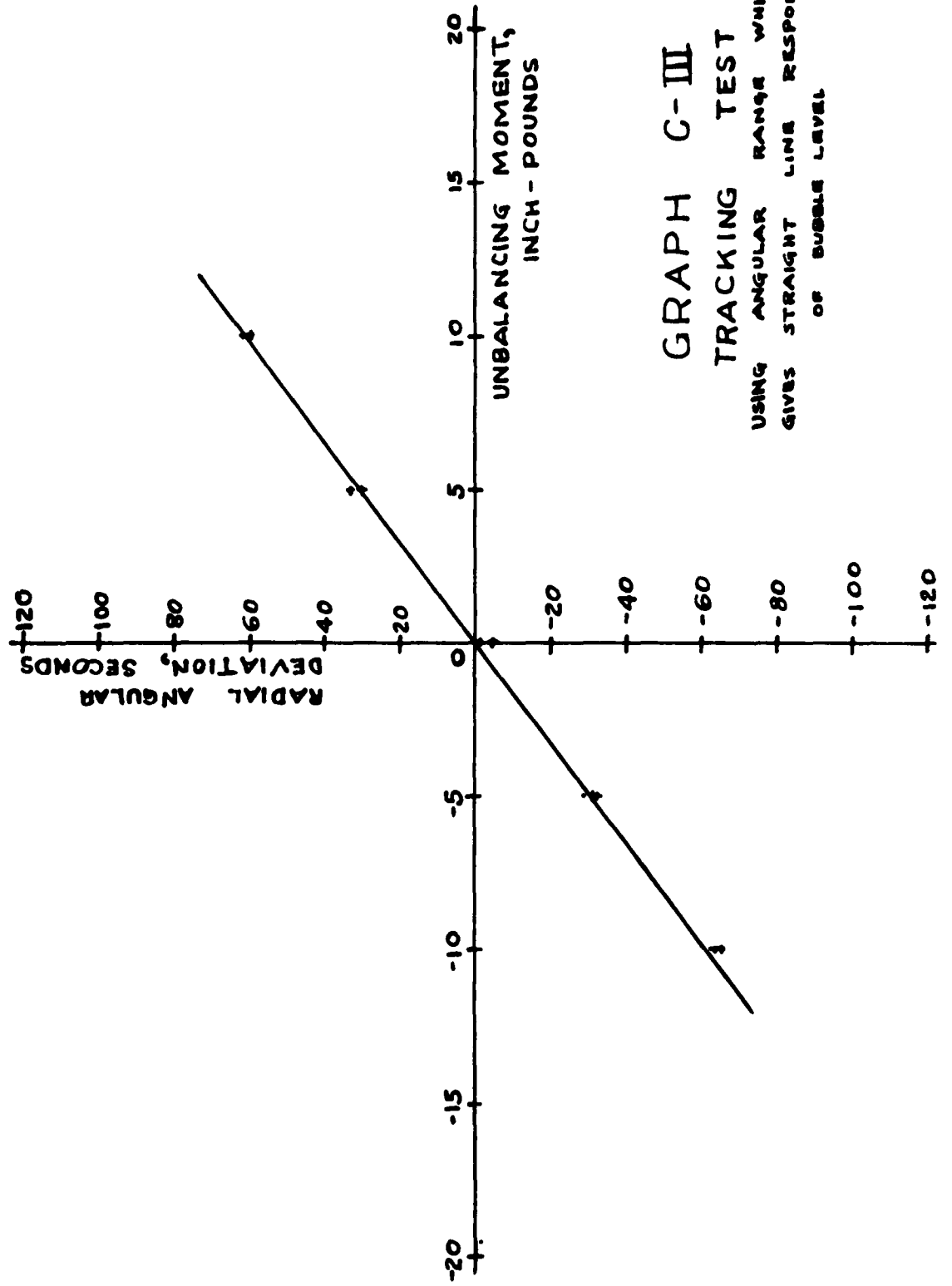
|  | $\overline{X}$ | $\overline{R}$ |
|---|---|---|
| Littlejohn T54 | $\pm.500"$ | .100" |
| Honest John M38 | $\pm 2.000"$ | .125" |

Therefore the instrument is more than adequately sensitive to measure these quantities.

A series of readings were taken to verify response characteristics of the instrument. When disparate results were obtained for longitudinal (5.5 sec/in.lb.) and radial (6.5 sec/in.lb.) sensitivities, the longitudinal indicating level was removed and placed parallel to the radial indicating level. A series of readings were taken with small offsetting moments both to right and to left. This verified the suspected cause of the trouble, i.e., the two levels did not read alike. The magnitude of difference is not such as to cause any difficulty in operation of the instrument.
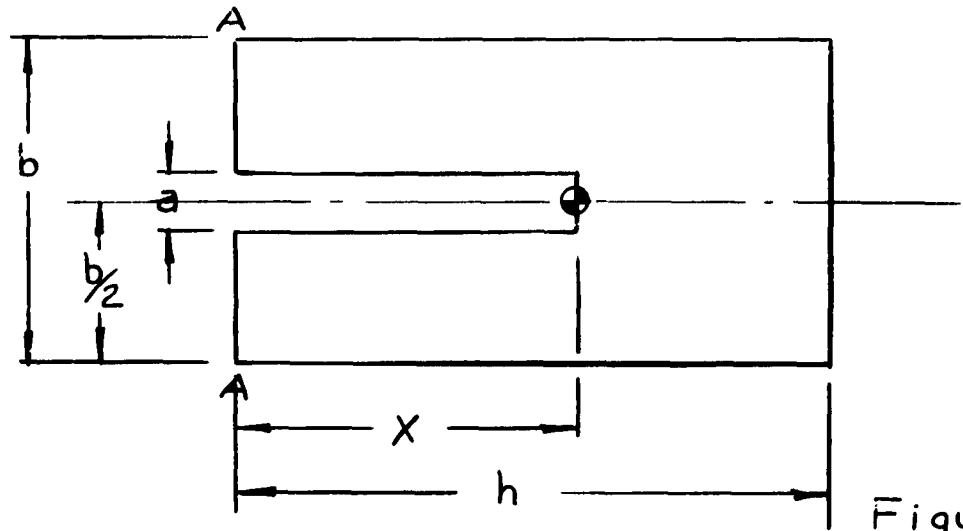
RADIAL ANGULAR DEVIATION, SECONDS

UNBALANCING MOMENT, INCH-POUNDS

INITIAL LOAD +100 IN.-LB.

INITIAL LOAD -100 IN.-LB.

GRAPH C-I
TRACKING TEST
ORIGINAL PIVOT SUPPORT

C-3

RADIAL ANGULAR DEVIATION, SECONDS

UNBALANCING MOMENT, INCH - POUNDS

INITIAL LOAD +100 IN. LB.

INITIAL LOAD, -100 IN. LB.

GRAPH C-II
TRACKING TEST

FLAT PIVOT SUPPORT,
HARDNESS ROCKWELL C-64

120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120

-20  -15  -10  -5  0  5  10  15  20

C-4

UNBALANCING MOMENT, INCH-POUNDS

RADIAL ANGULAR DEVIATION, SECONDS

GRAPH C-III
TRACKING TEST

USING ANGULAR RANGE WHICH
GIVES STRAIGHT LINE RESPONSE
OF BUBBLE LEVEL

C-5

## DESIGN OF LONGITUDINAL WEIGHTS



Figure D-

Taking the moment of the centroid of the areas with respect to axis ∆-A

1. $bh(\frac{h}{2})-ax(\frac{x}{2}) = (bh-ax)(x)$

Equation 1 defines the depth of slot, x, such that the centroid of the area is at the bottom edge of the slot, as shown. The values of a, b, and h are selected by design considerations, and are treated as constants.

2. The practical solution for the quadratic in x is given by

$$x = \frac{h[b- \sqrt{b(b-a)}]}{a}$$

For a one pound weight, the thickness, t, is calculated from

3. $t = \frac{1}{\rho(bh-ax)}$

where $\rho$ = density of the material

x = value from equation 2

The thickness of any other weight (W) is merely Wt.

## GENERAL PROCEDURE FOR CALCULATING PIVOT SHIFT

The basic equation used is derived from Diagram A1.

Equation 1

$$W_1(40.781-k)+W(\overline{X}-k) = W_F(\overline{X}_F+k)+W_2(k+23.219) \qquad (M_v = 0)$$

Take minimum values for warhead weight, W, and warhead CG location, $\overline{X}$. For perfect balance with these values, $W_1 = W_2 = 0$, and the equation reduces to

Equation 2

$$W(\overline{X}-k) = W_F(\overline{X}_F+k)$$

Substitute all known values and determine k.

Using this value of k and maximum values of W and $\overline{X}$, determine maximum $W_2$.

If this value of $W_2$ exceeds the total of weights on hand, recalculate k as follows:

A. Set up two simultaneous equations for the moments which exist at equilibrium when the warhead moment ($W\overline{X}$) is (1) maximum and (2) minimum.

B. Let the balancing weights be equal in each case, but of opposite direction. $(W_M)$.

C. Solve for k.

D. Determine $W_M$ by substituting maximum $W\&\overline{X}$ in A(1) above.

The ultimate limit of the instrument is established by the total weight which can fit between the flanges of the I-beam, or 166 lbs. For typical calculations, see Examples I through VI following.

DERIVATION OF MINIMUM AVAILABLE k OF INSTRUMENT

AND RELATED INSTRUMENT CONSTANTS

1. Correction in k to account for tapered gap:



Figure E-1
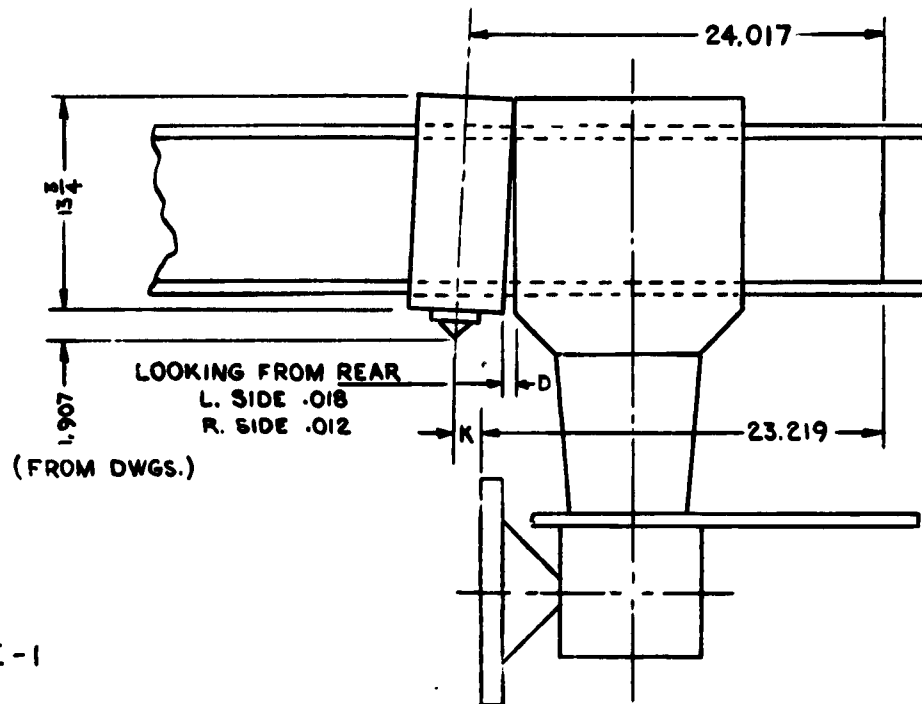
Average clearance $= \dfrac{.018 + .012}{2} = .015''$

Projected to pivot point, $\dfrac{.015 \times (13.75 + 1.907)}{13.75} = .017''$

Design value of $k_0$ (verified by parts inspection) $= .781$

Corrected $k_0 = .781 + .017 = .798''$

$S_2 = 24.000$ (design) $+ .017 = 24.017''$

$S_1 = 64.000 - 24.017'' = 39.983''$

Faceplate to CG of rear weight $= 24.017 - .798 = 23.219''$

Faceplate to CG of front weight $= 39.983 + .798 = 40.781$

Example I

Warhead Assembly M1A2, HE, Honest John (Dwg. FXP-107671)

$W \min = 1657-12 = 1645$ lb.

$\bar{X}$ nominal = Sta. 115.000 - Sta. 89.160 = 25.840

$\bar{X} \min = 25.84-.5 = 25.34''$

$W(\bar{X}-k) = W_F(\bar{X}_F+k)$     (Equation 2, p E-1)

$1645(25.34-k) = 1693(4.389+k)$

$k = 10.262''$

$W \max = 1669$      $\bar{X} \max = 26.34$      $W_1 = 0$

$W(\bar{X}-k)-W_F(\bar{X}_F+k) = W_2(k+23.219)$     (Equation 1, p E-1)

$1669(26.34-10.26)-1693(4.39+10.26) = W_2(10.262+23.219)$

$W_2 = 60.784$

Since 116.8 lbs calibrated weight is available, this is well within equipment capability.

Example II

Warhead M38, M144, M132, Honest John

$$W \text{ min} = 1625-12 = 1613$$

$$\bar{X} \text{ min} = 35-2 = 33$$

$$W(\bar{X}-k) = W_p(\bar{X}_p+k) \qquad \text{(Equation 2, p E-1)}$$

$$1635(33-k) = 1693(4.389+k)$$

$$k = 13.922$$

$$W \text{ max} = 1625+12 = 1637 \qquad W_1 = 0$$

$$\bar{X} \text{ max} = 35+2 = 37$$

$$W(\bar{X}-k)-W_p(\bar{X}_p+k) = W_2(k+23.219) \qquad \text{(Equation 1, p E-1)}$$

$$1637(37-13.9)-1693(4.389+13.9) = W_2(13.9+23.219)$$

$$W_2 = 184 \text{ lb.}$$

Since this exceeds the 116.8 lbs available, determine best k by simultaneous equation method described in Appendix E-1. Appendix E-5, following, illustrates the method by an example.

Example III

## PIVOT SHIFT TO ACCOMODATE

### HONEST JOHN WARHEAD, M38, M144, M132

$(W = 1625 \pm 12 \text{ lbs}, \bar{X} = 35 \pm 2 \text{ in.})$

Steps A & B (See p E-1)

I $\quad W_1(40.781-k)+W(\bar{X}-k)-W_F(\bar{X}_F+k)-W_2(k+23.219) = 0$

When $W$ = max value = 1637 lb and $\bar{X}$ = max value = 37", then

$W_1 = 0$ and $W_2$ = max value = $W_M$.

$1637(37-k)-1693(4.389+k)-W_M(23.219+k) = 0$

Ia $\quad 53137-3330k-W_M(23.219+k) = 0$

When $W$ = min value = 1613 lbs and $\bar{X}$ = min value = 33", then

$W_1$ = max value = $W_M$ and $W_2 = 0$

$W_M(40.781-k)+1613(33-k)-1693(4.389+k) = 0$

Ib $\quad 45797-3306k+W_M(40.781-k) = 0$

Multiply Ia by $(40.781-k)$ and Ib by $(23.219+k)$ and add to obtain

$k^2-9162.6k+134,595.3 = 0$

$k = \dfrac{9162.600 \pm 9133.176}{2}$

$k = 14.712$ (Here we utilize the answer resulting from the negative second term in the numerator. The answer resulting from the positive second term in the numerator is rejected as being physically inapplicable, although mathematically correct).

Substituting k in Ia, we obtain

$W_M = 109.17$ lbs, less than 116.8 lbs available, and therefore satisfactory.

Pivot shift = $k-k_o = 14.712-.798$

$\qquad = 13.914"$

Measurement of new pivot location is made between lower forward edge of main support cage and rear edge of pivot cage (at left, looking from rear of instrument):

$d = 13.914+.018 = 13.932"$ （See Figure E-1)

Example IV

## CALCULATION OF k FOR WARHEAD ASSEMBLY
## XM46, HE, HONEST JOHN
### (Drawing FXP-107672)

W min = 1657-12 = 1645 lb.

$\overline{X}$ nominal = Sta. 115.000 - Sta. 79.715 = 35.285"

$\overline{X}$ min = 35.285-.500 = 34.785"

$W(\overline{X}-k) = W_F(\overline{X}_F+k)$     (Equation 2, p E-1)

1645(34.785-k) = 1693(4.389+k)

k = 14.915"


W max = 1657+12 = 1669 lb.     $W_1$ = 0

$\overline{X}$ max = 35.285+.500 = 35.785"

$W(\overline{X}-k)-W_F(\overline{X}_F+k) = W_2(k+23.219)$     (Equation 1, p E-1)

1669(20.869)-1693(19.305) = $W_2$(38.135)

$W_2$ = 56.3 lb.


d = 14.916-.798+.018

  = 14.916-.780

  = 14.136"     (See Figure E-1)

Example V

## CALCULATION OF CG LOCATION FOR

### INERT LOADED WARHEAD ASSEMBLY M1A2, HE, HONEST JOHN

Measured weight, $W = 1687$ lb (design max. $= 1669$)

Nominal design $\bar{X} = 25.84''$

$$W(\bar{X}-k) = W_F(\bar{X}_F+k)$$

$$1687(25.84-k) = 1693(4.389+k)$$

$$k = 10.699$$

Desired Measurement $= k-.780 = 10.699-.780$

$$= 9.919$$

Actual Space Measured $= 9.957$

Actual $\underline{k} = 9.957+.780 = \underline{10.737}$

Pivot to rear balancing weight $= 10.737+23.219 = 33.956$

Pivot to front balancing weight $= 40.781-10.737 = 30.044$

$$W_1(30.044)+W(\bar{X}-10.737)-W_F(4.389+10.737)-W_2(33.956)-M_v = 0$$

$$\bar{X} = \frac{W_F(15.126)+W_2(33.956)+M_v-W_1(30.044)}{W} +10.737$$

$$\bar{X} = \frac{1693(15.126)+16,006(33.956)+1.5-.248(30.044)}{1687} +10.737$$

$\bar{X} = 26.235$

Design max $\bar{X} = 25.84+.50 = 26.34''$

CG longitudinal location is within tolerance.

Example V(a)

## CALCULATIONS FROM ADDITIONAL DATA FOR

### INERT LOADED WARHEAD ASSEMBLY M1A2, HE, HONEST JOHN

$W_1 = .248$ lbs $\qquad\qquad M_1 = 7.45$

$W_2 = 16.006$ lbs $\qquad\qquad M_2 = 543.50$

$M = 3$ (min) to $11.3$ (max), $6.5$ (avg)

See previous calculation for this warhead

$$\bar{X} = \frac{25608.30+543.5+6.5-7.45}{1687} +10.737$$

$\bar{X} = 26.238$

Max uncertainty in $\bar{X} = \frac{11.3-6.5}{1687} = \frac{4.8}{1687} = .0028"$

$\bar{R}$:

Data: $M_A = 54$

$\qquad M_B = -38.5$

$\qquad M_C = 14$

$\qquad M_D = 117$

$\qquad \frac{M_C-M_A}{2} = -20 \qquad \frac{M_B-M_D}{2} = -77.75$

$\qquad M_{resultant} = 80.4$

$\qquad \bar{R} = \frac{M_{resultant}}{W} = \frac{80.4}{1687}$

$\qquad \bar{R} = .0477"$

Example VI

## ILLUSTRATION OF METHOD OF CALCULATION
## WHERE SPECIAL ADAPTOR IS REQUIRED

Determination of CG location of Inert Loaded T-54 Littlejohn Warhead
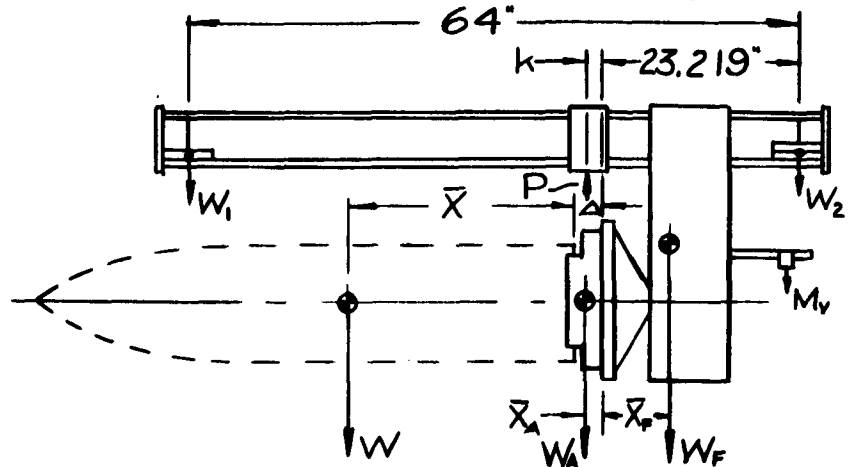


The general equation is:

$$\Sigma M_p = W(\bar{X}+\Delta-k)+W_1(40.781-k)-W_A(k-\bar{X}_A)-W_F(\bar{X}_F+k)-W_2(23.219+k)-M_v = 0$$

$$\Sigma M_p = W(\bar{X}+\Delta-.798)+39.983W_1-W_A(.798-\bar{X}_A)-W_F(\bar{X}_F+.798)-24.017W_2-M_v = 0$$

$\Delta = 1.264"$ (From Dwg. 8849043 and inspection report of adaptor dimensions)

$W_A = 92.7$ lbs

$\bar{X}_A = .466"$

$W_F = 1693$ lbs

$\bar{X}_F = 4.389"$

$\Delta$ is the distance from the faceplate to the warhead reference surface as determined by the adaptor.

$$W(\bar{X}+.466)+39.983W_1-92.7(.333)-1693(5.187)-24.017W_2-M_v = 0$$

$$\bar{X} = \frac{8812.461+24.017W_2+M_v-39.983W_1}{W} - .466$$

Set up charts to read $M_2 = 24.017W_2$

$$M_1 = 39.983W_1$$

Then let $M = M_2+M_v-M_1$ and $\bar{X} = \frac{8812.461+M}{W} - .466$

$W_2 = 7.001$　　　　　　　　　　　　$M_2 = 168.14$

　　　　　　　　　　　　　　　　　　　$M_v = \underline{16.10}$

　　　　　　　　　　　　　　　　　　　　　$184.24$

$W_1 = 61.006$　　　　　　　　　　　$M_1 = \underline{2439.20}$

$M = M_2 + M_v - M_1$　　　　　　　$M = -2254.96$

$W = 253$ (this unit was missing 2 flashcharges, thus was underweight)

$\bar{X} = \dfrac{8812.46 - 2254.96}{253} = .466$

$\bar{X} = 25.453''$

Establishment of Limits of Acceptance For Longitudinal Location of Warhead CG

I.  T54, M146, M8 Littlejohn

   From APPENDIX A-2

   $$\bar{X} = \frac{8812.461 + M}{W} - .466$$

   $$M = W(\bar{X} + .466) + 8812.461$$

   The following values were used to construct Chart E-I

   | W | $\bar{X}$ | M |
   |---|---|---|
   | 258 | 25.9 | -2010.05 |
   | 258 | 26.9 | -1752.05 |
   | 266 | 25.9 | -1799.12 |
   | 266 | 26.9 | -1533.12 |

II.  M38, M144, M132 Honest John

   From APPENDIX A-2

   $$\bar{X} = \frac{32337.993 + M}{W} + 14.712$$

   $$M = W(\bar{X} - 14.712) - 32337.993$$

   The following values were used to construct Chart E-II

   | W | $\bar{X}$ | M |
   |---|---|---|
   | 1637 | 33 | -2400.03 |
   | 1637 | 37 | +4147.97 |
   | 1613 | 33 | -2838.94 |
   | 1613 | 37 | +3613.06 |

APPENDIX E-10

TABLE E-1

CONVERSION TABLES

Balancing Weights to Longitudinal Moments

| Weight | k—.798 | | k—10.262 | | k—14.712 | | k—14.916 | |
|--------|--------|--------|----------|----------|----------|----------|----------|----------|
| | $M_1$ | $M_2$ | $M_1$ | $M_2$ | $M_1$ | $M_2$ | $M_1$ | $M_2$ |
| .248 | 9.916 | 5.956 | 7.569 | 8.303 | 6.465 | 9.407 | 6.415 | 9.457 |
| 1.001 | 40.023 | 24.041 | 30.550 | 33.514 | 26.095 | 37.969 | 25.891 | 38.173 |
| 2.000 | 79.966 | 48.034 | 61.038 | 66.963 | 52.138 | 75.862 | 51.730 | 76.270 |
| 3.001 | 119.989 | 72.075 | 91.588 | 100.476 | 78.233 | 113.831 | 77.621 | 114.443 |
| 3.988 | 159.452 | 95.780 | 121.710 | 133.522 | 103.963 | 151.269 | 103.150 | 152.082 |
| 5.001 | 199.955 | 120.109 | 152.626 | 167.438 | 130.371 | 189.693 | 129.351 | 190.713 |
| 10.004 | 399.990 | 240.266 | 305.312 | 334.944 | 260.794 | 379.462 | 258.753 | 381.503 |
| 19.925 | 796.661 | 478.539 | 608.091 | 667.109 | 519.425 | 755.775 | 515.360 | 759.840 |
| 20.625 | 824.649 | 495.351 | 629.454 | 690.546 | 537.673 | 782.327 | 533.466 | 786.534 |
| 51.002 | 2039.213 | 1224.915 | 1556.530 | 1707.598 | 1329.571 | 1934.557 | 1319.167 | 1944.961 |

E-12

CHART E-1

LONGITUDINAL
CENTER OF GRAVITY

LITTLEJOHN

BALANCING MOMENT, HUNDREDS OF INCH-POUNDS

E-13

CHART E-II

LONGITUDINAL
CENTER OF GRAVITY

HONEST JOHN WARHEAD
M39, M52, M144

INCHES

X, NET MOMENT, THOUSANDS OF INCH-POUNDS
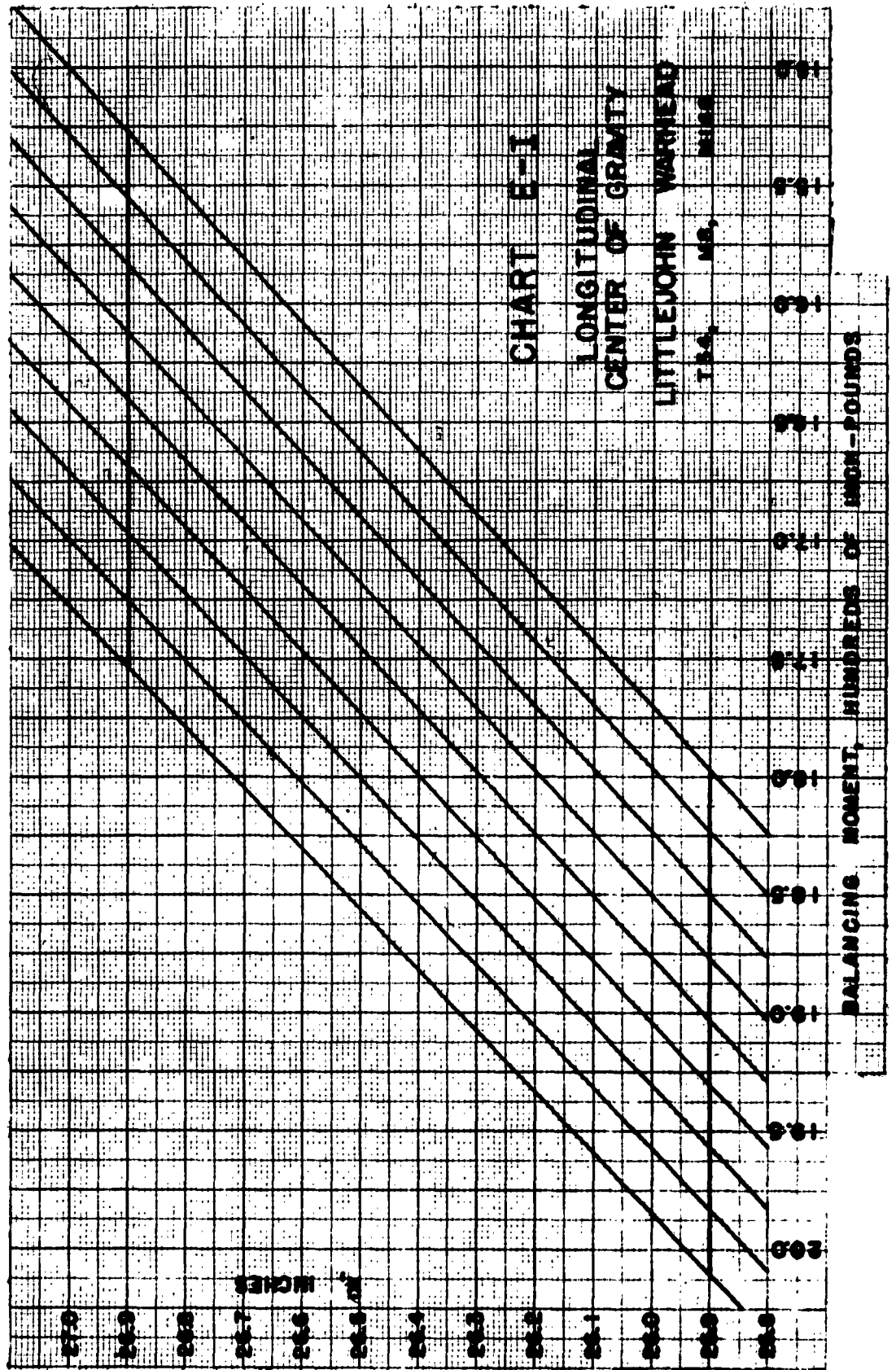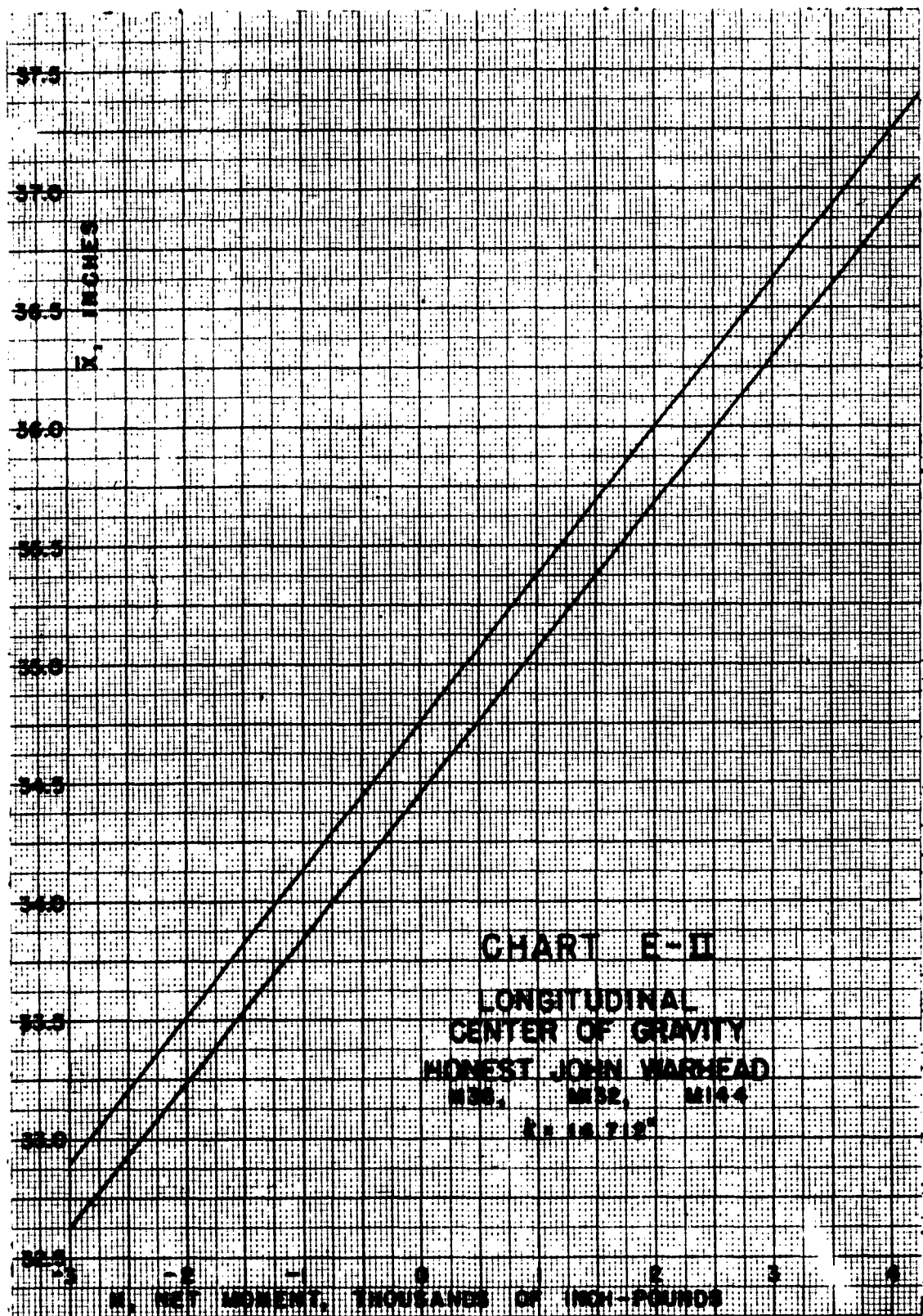
## DETERMINATION of $X_F$, USING STANDARD
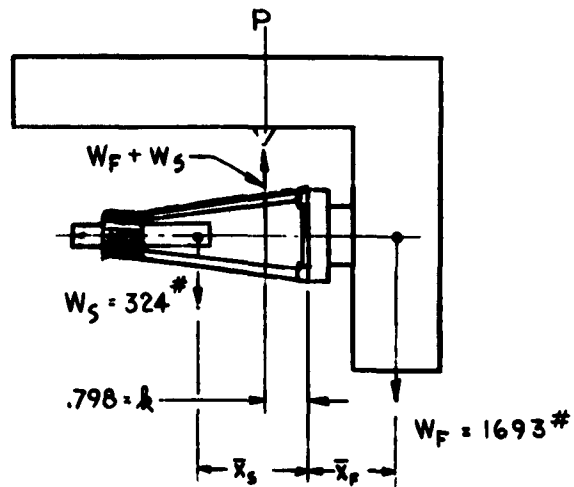


$$\bar{X}_S = 27.900^{\pm.002}$$

At balance,

$$\Sigma M_{(P)} = 0 = W_F (\bar{X}_F + .798) = W_S (\bar{X}_S - .798)$$

$$\bar{X}_F = \frac{W_S (\bar{X}_S - .798)}{W_F} - .798$$

$$= \frac{324 (27.102)}{1693} - .798$$

$$\bar{X}_F = 4.389 \text{ inches}$$

## DISTRIBUTION LIST

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Picatinny Arsenal, Dover, New Jersey

*A UNIQUE UNIVERSAL TYPE INSTRUMENT TO LO-CATE CENTER OF GRAVITY OF VARIOUS WAR-HEADS*

*David Stein, Mark H. Weinberg*

Technical report 3059, December 1962, 65 pp, photographs, diagrams, graphs, tables. Unclassified report from the Test Methods and Equipment Section of the Quality Assurance Division.

A unique, universal type device to locate the center of gravity of various warheads is described. Simple basic principles are utilized in a new configuration to produce a highly precise instrument which may be adapted to locate center of gravity of warheads with a constant accuracy of

(over)

1. Center of Gravity
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about 0.002 inch. Both longitudinal distance from warhead reference surface and radial displacement from warhead axis are determined in a single handling of the warhead, thereby eliminating the need for two separate devices.

The instrument was designed specifically for warheads in the Littlejohn and Honest John Systems (weight range up to 1700 pounds; centroid up to 37 inches from rear mounting surface). However, its total range of capability extends from essentially zero to a warhead which exerts 80,000 inch-pounds of moment (warhead weight times centroidal distance from rear reference plane).

Mounting provisions are made for the following items of current production: Littlejohn Warheads M146 and M8, and Honest John Warheads M38, M132, and M144.

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